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1. The Problem

Let me start with a well-known story. Kant held that logic and conceptual analysis alone cannot account for our knowledge of arithmetic: "however we might turn and twist our concepts, we could never, by the mere analysis of them, and without the aid of intuition, discover what is the sum [7+5]" (*KrV*, B16). Frege took himself to have shown that Kant was wrong about this. According to Frege's *logicist thesis*, every arithmetical concept can be defined in purely logical terms, and every theorem of arithmetic can be proved using only the basic laws of logic. Hence, Kant was wrong to think that our grasp of arithmetical concepts and our knowledge of arithmetical truth depend on an extralogical source—the pure intuition of time (Frege 1884, §89, §109). Arithmetic, properly understood, is just a part of logic.

Never mind whether Frege was right about this. I want to address a different question: Does Frege's position on arithmetic really contradict Kant's? I do not deny that Frege endorsed

(F) Arithmetic is reducible to logic

or that Kant endorsed

(K) Arithmetic is not reducible to logic.[1]

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[1] In what follows, when I use the term 'logic' in connection with Kant, I will mean what he calls 'pure general logic' (*KrV*, A55/B79), as opposed to 'special,' 'applied', or 'transcendental' logics. (Kant often uses 'logic' in this restricted sense: e.g., *KrV*, B ix, A61/B86, A598/B626; *JL*, 13.) In denying that arithmetic is analytic, Kant is denying that it is reducible to *pure general* logic and definitions. (Analytic truths are knowable through the principle of contradiction, a principle of pure general logic (*KrV*, A151/B190).) Similarly, the "logic" to which Frege claims to reduce arithmetic is pure (independent of human psychology (1893, xvii)) and general (unrestricted in its subject matter (1884, iii–iv)). So in assessing Frege's claim to be contradicting Kant's view, it is appropriate to restrict our attention to pure general logic.

But (F) and (K) are contradictories only if 'logic' has the same sense in both. And it is not at all clear that it does.

First, the resources Frege recognizes as logical far outstrip those of Kant's logic (Aristotelian term logic with a simple theory of disjunctive and hypothetical propositions added on). The most dramatic difference is that Frege's logic allows us to define concepts using nested quantifiers, while Kant's is limited to representing inclusion relations.[2] For example, using Fregean logic (in modern notation) we can say that a relation R *is a dense ordering just in case*

(D) $(\forall x)(\forall y)(Rxy \supset (\exists z)(Rxz \;\&\; Rzy))$

But, as Friedman (1992) has emphasized, we cannot express this condition using the resources of Kant's logic.[3] For Kant, the only way to represent denseness is to model it on the infinite divisibility of a line in space. As Friedman explains, "denseness is represented by a definite fact about my intuitive capacities: namely, whenever I can represent (construct) two distinct points *a* and *b* on a line, I can represent (construct) a third point *c* between them" (64). What Kant can represent only through construction in intuition, Frege can represent using vocabulary he regards as logical. And quantifier dependence is only the tip of the iceberg: Frege's logic also contains higher-order quantifiers and a logical functor for forming singular terms from open sentences. Together, these resources allow Frege to define many notions that Kant would not have regarded as expressible without construction in pure intuition: infinitude, one-one correspondence, finiteness, natural number, and even individual numbers.

It is natural for us to think that Frege refuted Kant's view that the notion of a dense ordering can only be represented through construction in intuition. Surely, we suppose, if Kant had been resurrected, taught modern logic, and confronted with (D), he would have been rationally compelled to abandon this view. But this is far from clear. It

[2] Frege calls attention to this difference in 1884, §88.

[3] That is, we cannot express it in a way that would allow us to infer from it, using logic alone, the existence of as many objects as we please. If we start with the categorical propositions 'Every pair of rational numbers is a pair of rational numbers with a rational number between them' and '<*A*, *B*> is a pair of rational numbers', then we can infer syllogistically '<*A*, *B*> is a pair of rational numbers with a rational number between them'. But Kant's logic contains no way to move from this proposition to the explicitly existential categorical proposition 'Some rational number is between *A* and *B*'. There is no common "middle term."

would have been open to Kant to claim that Frege's Begriffsschrift is not a proper logic at all, but a kind of abstract combinatorics, and that the meaning of the iterated quantifiers can only be grasped through construction in pure intuition.[4] As Dummett observes, "It is ... not enough for Frege to show arithmetic to be constructible from some arbitrary formal theory: he has to show that theory to be logical in character, and to be a correct theory of logic" (1981, 15). Kant might have argued that Frege's expansion of logic was just a change of subject, just as Poincaré charged that Russell's "logical" principles were really intuitive, synthetic judgments in disguise:

> We see how much richer the new logic is than the classical logic; the symbols are multiplied and allow of varied combinations which are no longer limited in number. Has one the right to give this extension to the meaning of the word *logic*? It would be useless to examine this question and to seek with Russell a mere quarrel about words. Grant him what he demands, but be not astonished if certain verities declared irreducible to logic in the old sense of the word find themselves now reducible to logic in the new sense—something very different.
>
> We regard them as intuitive when we meet them more or less explicitly enunciated in mathematical treatises; have they changed character because the meaning of the word logic has been enlarged and we now find them in a book entitled *Treatise on Logic*? (Poincaré 1908, 461)

Hao Wang sums up the situation well:

> Frege thought that his reduction refuted Kant's contention that arithmetic truths are synthetic. The reduction, however, cuts both ways. ... if one believes firmly in the irreducibility of arithmetic to logic, he will conclude from Frege's or Dedekind's successful reduction that what they take to be logic contains a good deal that lies outside the domain of logic. (1957, 80)

We're left, then, with a dialectical standoff: Kant can take Frege's proof that arithmetical concepts can be expressed in his Begriffsschrift as a demonstration that the Begriffsschrift is not entirely logical in character.

A natural way to resolve this standoff would be to appeal to a shared characterization of logic. By arguing that the Begriffsschrift fits a characterization of logic that Kant accepts, Frege could blunt one edge of

[4] This line is not so implausible as it may sound. For consider how Frege explains the meaning of the (iterable) quantifiers in the *Begriffsschrift* by appealing to the substitution of a potentially infinite number of expressions into a linguistic frame (Frege 1879). This is not the only way to explain the meaning of the quantifiers, but other options (Tarski 1933, Beth 1961) also presuppose a grasp of the infinite.

Wang's double-edged sword. Of course, it is not true *in general* that two parties who disagree about what falls under a concept *F* must be talking past each other unless they can agree on a common definition or characterization of *F*. We mean the same thing by 'gold' as the ancient Greeks meant by '*chrusos*', even though we characterize it by its microstructure and they by its phenomenal properties, for these different characterizations (in their contexts) pick out the same "natural kind" (Putnam 1975). And it is possible for two parties to disagree about the disease arthritis even if one defines it as a disease of the joints exclusively, while the other defines it as a disease of the joints and ligaments, for there are experts about arthritis to whom both parties defer in their use of the word (Burge 1979). But 'logic' does not appear to be a "natural kind" term. Nor are there experts to whom both parties in this dispute might plausibly defer. (No doubt Frege and Kant would each have regarded *himself* as an expert on the demarcation of logic, and neither would have deferred to the other.) Thus, unless Kant and Frege can agree, in general terms, about what logic *is*, there will be no basis (beyond the contingent and surely irrelevant fact that they use the same *word*) for saying that they are disagreeing about a single subject matter, *logic*, as opposed to saying compatible things about two subject matters, $logic_{Frege}$ and $logic_{Kant}$.

But there is a serious obstacle in the way of finding a shared general characterization. The difficulty is that Frege rejects one of Kant's most central views about the nature of logic: his view that logic is purely *Formal*.[5] According to Kant, pure general logic (henceforth, 'logic')[6] is distinguished from mathematics and the special sciences (as well as from special and transcendental logics) by its complete abstraction from semantic content:

> General logic abstracts, as we have shown, from all content of cognition, i.e. from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another, i.e., the form of thinking in general. (*KrV*, A55/B79; cf. A55/B79, A56/B80, A70/B95, A131/B170; *JL*, 13, §19).

To say that logic is Formal, in this sense, is to say that it is completely indifferent to the semantic contents of concepts and judgments and

[5] There are many senses in which logic might be called "formal" (see MacFarlane 2000): I use the capitalized 'Formal' to mark out the Kantian usage (to be elaborated below).

[6] See note 1, above.

attends only to their forms. For example, in dealing with the judgment that some cats are black, logic abstracts entirely from the fact that the concept *cat* applies to cats and the concept *black* to black things, and considers only the way in which the two concepts are combined in the thought: the judgment's *form* (particular, affirmative, categorical, assertoric) (*KrV*, A56/B80; *JL*, 101). Precisely because it abstracts in this way from that by virtue of which concepts and judgments are *about* anything, logic can yield no extension of knowledge about reality, about objects:

> since the mere form of cognition, however well it may agree with logical laws, is far from sufficing to constitute the material (objective) truth of the cognition, nobody can dare to judge of objects and to assert anything about them merely with logic. (*KrV*, A60/B85)

This picture of logic is evidently incompatible with Frege's view that logic can supply us with substantive knowledge about objects—for example, the natural numbers (1884, §89).

But Frege has reasons for rejecting it that are independent of his commitment to logicism and logical objects: on his view, there are certain *concept* and *relation* expressions from whose content logic cannot abstract. If logic were "unrestrictedly formal," he argues,

> then it would be without content. Just as the concept *point* belongs to geometry, so logic, too, has its own concepts and relations; and it is only in virtue of this that it can have a content. *Toward what is thus proper to it, its relation is not at all formal.* No science is completely formal; but even gravitational mechanics is formal to a certain degree, in so far as optical and chemical properties are all the same to it. ... To logic, for example, there belong the following: negation, identity, subsumption, subordination of concepts. (1906, 428, emphasis added)

Whereas on Kant's view the 'some' in 'some cats are black' is just an indicator of form and does not itself have semantic content, Frege takes it (or rather, its counterpart in his Begriffsschrift) to have its own semantic content, to which logic must attend.[7] The existential quantifier refers to a second-level concept, a function from concepts to truth values. Thus logic, for Frege, cannot abstract from all semantic content: it must attend, at least, to the semantic contents of the logical expressions, which on Frege's view function semantically just like non-

[7] I do not claim that Frege was always as clear about these issues as he is in Frege 1906. For an account of his progress, see chapter 5 of MacFarlane 2000.

logical expressions.[8] And precisely because it does not abstract from these contents, it can tell us something about the objective world of objects, concepts, and relations, and not just about the "forms of thought."

In view of this major departure from the Kantian conception of logic, it is hard to see how Frege can avoid the charge of *changing the subject* when he claims (against Kant) that arithmetic has a purely "logical" basis. To be sure, there is also much in common between Frege's and Kant's characterizations of logic. For example, as I will show in section 2, both think of logic as providing universally applicable norms for thought. But if Formality is an essential and independent part of Kant's characterization of logic, then it is difficult to see how this agreement on logic's universal applicability could help. Kant could agree that Frege's Begriffsschrift is universally applicable but deny that it is *logic,* on the grounds that it is not completely *Formal.* For this reason, attempts to explain why Frege's claim contradicts Kant's by invoking shared characterizations of logic are inadequate, as long as the disagreement on Formality is left untouched. They leave open the possibility that 'logic' in Kant's mouth has a strictly *narrower* meaning than 'logic' in Frege's mouth—narrower in a way that rules out logicism on broadly conceptual grounds.

Though I have posed the problem as a problem about Kant and Frege, it is equally pressing in relation to current discussions of logicism. Like Kant, many contemporary philosophers conceive of logic in a way that makes Fregean logicism look *incoherent.* Logic, they say, cannot have an ontology, cannot make existence claims. If this is meant as a quasi-analytic claim about logic (as I think it usually is),[9] then Frege's project of grounding arithmetic in pure logic is hopeless from the start.

[8] For example, both the logical expression '... = ...' and the nonlogical expression '... is taller than ...' refer to two-place relations between objects. They differ in *what* relations they refer to, but there is no *generic* difference in their semantic function. Similarly, both 'the extension of ...' and 'the tallest ...' refer to functions from concepts to objects. A Fregean semanticist doesn't even need to know which expressions are logical and which nonlogical (unless it is necessary to define logical independence or logical consequence; cf. Frege 1906).

[9] Surely it is not a *discovery* of modern logic that logic cannot make existence claims. What technical result could be taken to establish this? Russell's paradox demolishes a certain way of working out the idea that logic alone can make existence claims, but surely it does not show that talk of "logical objects" is inevitably doomed to failure. Tarski's definition of logical consequence ensures that no logically true sentence can assert the existence of more than one object—logical truths must hold in arbitrary nonempty domains—but this is a *definition,* not a result. At best it might be argued that the *fruitfulness* of Tarski's definition proves its "correctness."

A number of philosophers have drawn just this conclusion. For example, Hartry Field rejects logicism on the grounds that logic, in "the normal sense of 'logic'," cannot make existence claims (1984, 510; not coincidentally, he cites Kant). Harold Hodes characterizes Frege's theses that (1) mathematics is really logic and (2) mathematics is about mathematical objects as "uncomfortable passengers in a single boat" (1984, 123). And George Boolos claims that in view of arithmetic's existential commitments, it is "trivially" false that arithmetic can be reduced to logic:

> Arithmetic implies that there are two distinct numbers; were the relativization of this statement to the definition of the predicate "number" provable by logic alone, logic would imply the existence of two distinct objects, which it fails to do (on any understanding of logic now available to us). (1997, 302)

All three of these philosophers seem to be suggesting that Frege's logicism can be ruled out from the start on broadly *conceptual* grounds: no system that allows the derivation of nontrivial existential statements can *count as* a logic.

If they are right, then we are faced with a serious historical puzzle: how could Frege (or anyone else) have thought that this conceptually incoherent position was worth pursuing? The question is not lost on Boolos:

> How, then ... *could* logicism ever have been thought to be a mildly plausible philosophy of mathematics? Is it not obviously demonstrably inadequate? How, for example, could the theorem
>
> $$\forall x(\neg x < x) \wedge \forall x \forall y \forall z(x < y \wedge y < z \rightarrow x < z) \wedge \forall x \exists y(x < y)$$
>
> of (one standard formulation of) arithmetic, a statement that holds in no finite domain but which expresses a basic fact about the standard ordering of the natural numbers, be even a "disguised" truth of logic? (Boolos 1987, 199–200)

Whereas Boolos leaves this question rhetorical, my aim in this paper is to answer it. In the process of showing how Frege can engage with Kant over the status of arithmetic, I will articulate a way of thinking about logic that leaves logicism a coherent position (though still one that faces substantial technical and philosophical difficulties). My strategy has two parts. First, in section 2, I show that Frege and Kant concur in characterizing logic by a characteristic I call its "Generality." This shared notion of Generality must be carefully distinguished from contemporary notions of logical generality (including invariance under permutations) that are sometimes mistakenly attributed to Frege. Second, in section 3, I argue that Formality is not, for Kant, an independent defining feature of logic, but rather a *consequence* of the Generality of logic, together with several auxiliary premises from Kant's critical philosophy. Since Frege rejects two of these premises on general philosophical grounds (as I show in section 4), he can coherently hold that Kant was *wrong* about the Formality of logic. In this way, the dispute between Kant and Frege on the status of arithmetic can be seen to be a substantive one, not a merely verbal one: Frege can argue that his Begriffsschrift is a *logic* in Kant's own sense.

2. Generality

It is uncontroversial that both Kant and Frege characterize logic by its maximal *generality*. But it is often held that Kant and Frege conceive of the generality of logic so differently that the appearance of agreement is misleading.[10] There are two main reasons for thinking this:

(1) For Kant, logic is canon of reasoning—a body of rules—while for Frege, it is a science—a body of truths. So it appears that the same notion of generality cannot be appropriate for both Kant's and Frege's conceptions of logic. Whereas a *rule* is said to be general in the sense of being generally *applicable*, a *truth* is said to be general in the sense of being *about* nothing in particular (or about everything indifferently).

(2) For Kant, the generality of logical laws consists in their abstraction from the content of judgments, while for Frege, the generality of logical laws consists in their unrestricted quantification over all objects and all concepts. Hence Kant's notion of generality makes it impossible for logical laws to have substantive content, while Frege's is consistent with his view that logical laws say something about the world.

Each of these arguments starts from a real and important contrast between Kant and Frege. But I do not think that these contrasts show that Kant and Frege *mean* something different in characterizing logic as maximally "general." The first argument is right to emphasize that Frege, unlike Kant, conceives of logic as a science, a body of truths. But (I will argue) it is wrong to conclude that Frege and Kant cannot use the same notion of generality in demarcating logic. For Frege holds

[10] See, for example, Ricketts 1985, 4–5; 1986, 80–82; Wolff 1995, 205–223.

that logic can be viewed both as a science and as a normative discipline; in its latter aspect it can be characterized as "general" in just Kant's sense. The second argument is right to emphasize that Kant takes the generality of logic to preclude logic's having substantive content. But (I will argue) the notion of generality Kant shares with Frege—what I will call 'Generality'—is not *by itself* incompatible with contentfulness. As we will see in section 3, the incompatibility arises only in the context of other, specifically Kantian commitments. Thus, the second argument is guilty of conflating Kant's distinct notions of Generality and Formality into a single unarticulated notion of formal generality.[11]

Descriptive Characterizations of the Generality of Logic

It is tempting to think that what Frege means when he characterizes logic as a maximally general science is that its truths are not about anything in particular. This is how Thomas Ricketts glosses Frege: "in contrast to the laws of special sciences like geometry or physics, the laws of logic do not mention this or that thing. Nor do they mention properties whose investigation pertains to a particular discipline" (1985, 4–5). But this is *Russell's* conception of logical generality, not Frege's.[12] For on Frege's mature view, the laws of logic *do* mention properties (that is, concepts and relations) "whose investigation pertains to a particular discipline": identity, subordination of concepts, and negation, among others.[13] Although these notions are *employed* in every discipline, only one discipline—logic—is charged with their investigation. This is why Frege explicitly *rejects* the view that "as far as logic itself is concerned,

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[11] On Michael Wolff's view, for example, 'formal logic' in Kant is *synonymous* with 'general pure logic' (1995, 205). This flattening of the conceptual landscape forces Wolff to attribute the evident differences in Kant's and Frege's conceptions of logic to differences in their concepts of logical generality.

[12] Compare this passage from Russell's 1913 manuscript *Theory of Knowledge*: "Every logical notion, in a very important sense, is or involves a *summum genus*, and results from a process of generalization which has been carried to its utmost limit. This is a peculiarity of logic, and a touchstone by which logical propositions may be distinguished from all others. A proposition which mentions any definite entity, whether universal or particular, is not logical: no one definite entity, of any sort or kind, is ever a constituent of any truly logical proposition" (Russell 1992, 97–98).

[13] It might be objected that logic is not a *particular* discipline; it is, after all, the most *general* discipline. But this just shifts the bump in the rug: instead of asking what makes logic "general," we must now ask what makes nonlogical disciplines "particular." It's essentially the same question. It might also be objected that identity, negation, and so on are only *used* in logic, not "mentioned." But this is a confusion. The *signs* for identity, negation, etc. are used, not mentioned—Frege's logic is not our metalogic—but these signs (on Frege's view) *refer to*

each object is as good as any other, and each concept of the first level as good as any other and can be replaced by it, etc." (1906, 427–28).

Still, it might be urged that these notions whose investigation is peculiar to logic are *themselves* characterized by their generality: their insensitivity to the differences between particular objects. Many philosophers and logicians have suggested, for example, that logical notions must be invariant under all permutations of a domain of objects,[14] and at least one (Kit Fine) has proposed that permutation invariance "is the formal counterpart to Frege's idea of the generality of logic" (1998, 556). But Frege could hardly have held that logic was general in *this* sense, either. If arithmetic is to be reducible to logic, and the numbers are objects, then the logical notions had better *not* be insensitive to the distinguishing features of objects. Each number, Frege emphasizes, "has its own unique peculiarities" (1884, §10). For example, 3, but not 4, is prime. If logicism is true, then, it must be possible to distinguish 3 from 4 using logical notions alone. But even apart from his commitment to logicism, Frege could not demarcate the logical notions by their permutation invariance. For he holds that *every sentence* is the name of a particular object: a truth value. As a result, not even the *truth functions* in his logic are insensitive to differences between particular objects: negation and the conditional must be able to distinguish the True from all other objects. Finally, every one of Frege's logical laws employs a concept, the "horizontal" (—), whose extension is {the True} (1893, §5). The horizontal is plainly no more permutation-invariant than the concept *identical with Socrates*, whose extension is {Socrates}.

It is a mistake, then, to cash out the "generality" of Frege's logic in terms of insensitivity to the distinguishing features of objects; this conception of generality is simply incompatible with Frege's logicism. How, then, should we understand Frege's claim that logic is characterized by its *generality*? As Hodes asks, "How can a part of logic be about a distinctive domain of objects and yet preserve its topic-neutrality" (1984, 123)?[15]

concepts and relations, which are therefore *mentioned*. It is hard to see how Frege could avoid saying that logic investigates the relation of identity (among others), in just the same way that geometry investigates the relation of parallelism (among others).

[14] See Mautner 1946; Mostowski 1957, 13; Tarski 1986; McCarthy 1981; van Benthem 1989; Sher 1991 and 1996; McGee 1996.

[15] See also Sluga 1980: "Among the propositions of arithmetic are not only those that make claims about all numbers, but also those that make assertions about particular numbers and others again that assert the existence of numbers. The question is how such propositions could be regarded as universal, and therefore logical, truths" (109).

A Normative Characterization of the Generality of Logic

I want to suggest that no *descriptive* characterization of generality can capture what Frege has in mind when he characterizes logic as general. The generality of logic, for Frege as for Kant, is a *normative* generality: logic is general in the sense that it provides constitutive norms for thought *as such*, regardless of its subject matter.[16]

But first we must get clear about the precise sense in which logical laws, for Frege, are normative. As Frege is well aware, 'law' is ambiguous: "In one sense a law asserts what is; in the other it prescribes what ought to be" (1893, xv). A *normative law* prescribes what one ought to do or provides a standard for the evaluation of one's conduct as good or bad. A *descriptive law*, on the other hand, describes certain regularities in the order of things—typically those with high explanatory value or counterfactual robustness. Are the laws of logic normative or descriptive, on Frege's view?

Both. Frege does not think that logical laws are prescriptive in their *content* (Ricketts 1996, 127). They have the form "such and such is the case," not "one should think in such and such a way":

> The word 'law' is used in two senses. When we speak of moral or civil laws we mean prescriptions, which ought to be obeyed but with which actual occurrences are not always in conformity. Laws of nature are general features of what happens in nature, and occurrences in nature are always in accordance with them. *It is rather in this sense that I speak of laws of truth* [i.e., laws of logic]. Here of course it is not a matter of what happens but of what is. (1918, 58)

Consider, for example, Basic Law IIa (1893, §19): in modern notation, $\forall F \forall x(\forall y \mathrm{F}(y) \supset \mathrm{F}(x))$. This is just a claim about all concepts and all objects, to the effect that if the concept in question holds of all objects, then it holds of the object in question. There are no *oughts* or *mays* or *musts*: no norms in sight![17]

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[16] 'Thought' is of course ambiguous between an "act" and an "object" interpretation. I am using it here (and throughout) in the "act" sense (as equivalent to 'thinking', that is, forming beliefs on the basis of other beliefs). The norms logic provides, on Frege's view, are ought-to-do's, not ought-to-be's. (See also note 18, below.)

[17] Of course there are also logical rules of inference, like modus ponens, and these have the form of permissions. As Frege understands them, they are genuine norms for inferring, not just auxiliary rules for generating logical truths from the axioms. But they are not norms for thinking as such: because they are specified syntactically, they are binding on one only insofar as one is using a particular formalized language. The rule for modus ponens in a system where the conditional is written '$\supset$' is different from the rule for modus ponens in a system where the conditional is written '$\rightarrow$'.

But Frege also says that logic, like ethics, can be called "a normative science" (1979, 128). For although logical laws are not prescriptive in their content, they *imply* prescriptions and are thus prescriptive in a broader sense: "From the laws of truth there follow prescriptions about asserting, thinking, judging, inferring" (1918, 58). *Because* the laws of logic are as they are, one ought to think in certain ways and not others. For example, one ought not believe both a proposition and its negation. Logical laws, then, have a dual aspect: they are descriptive in their content but imply norms for thinking.

On Frege's view, this dual aspect is not unique to laws of logic: it is a feature of *all* descriptive laws:

> Any law asserting what is, can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought. This holds for laws of geometry and physics no less than for laws of logic. The latter have a special title to the name 'laws of thought' only if we mean to assert that they are the most general laws, which prescribe universally the way in which one ought to think if one is to think at all. (1893, xv)

Frege's line of thought here is subtle enough to deserve a little unpacking. Consider the statement "the white King is at C3." Though the statement is descriptive in its content, it has prescriptive consequences in the context of a game of chess: for instance, it implies that white is prohibited from moving a bishop from C4 to D5 if there is a black rook at C5. Now instead of chess, consider the "game" of thinking about the physical world (not just grasping thoughts, but evaluating them and deciding which to endorse).[18] As in chess, "moves" in this game—judgments—can be assessed as correct or incorrect. Judgments about the physical world are correct to the extent that their contents match the physical facts. Thus, although the laws of physics are descriptive laws—they tell us about (some of) these physical facts—they have prescriptive consequences for anyone engaged in the "game" of thinking about the physical world: such a thinker *ought not* make judgments that are incompatible with them. Indeed, insofar as one's activity is to count as

[18] Frege often uses 'thinking' to mean *grasping* thoughts (1979, 185, 206; 1918, 62), but it is hard to see how the laws of logic could provide norms for thinking in this sense. The principle of non-contradiction does not imply that we ought not *grasp* contradictory thoughts: indeed, sometimes we *must* grasp such thoughts, when they occur inside the scope of a negation or in the antecedent of a conditional (1923, 50). Thus, it seems most reasonable to take Frege's talk of norms for thinking as talk of norms for *judging*. Norms for thinking, in this sense, will include norms for inferring, which for Frege is simply the making of judgments on the basis of other judgments.

making judgments about the physical world at all, it must be assessable for correctness in light of the laws of physics.[19] In this sense, the laws of physics provide *constitutive norms* for the activity of thinking about the physical world. Only by opting out of that activity altogether—as one does when one is spinning a fantasy tale, for example, or talking about an alternative possible universe—can one evade the force of these norms.

This is not to say that one cannot think *wrongly* about the physical world: one's judgments need not *conform to* the norms provided by the laws of physics; they need only be *assessable* in light of these norms. (Analogously, one can make an illegal move and still count as playing chess.) Nor is it to say that one must be *aware* of these laws in order to think about the physical world. (One can be ignorant of some of the rules and still count as playing chess.) The point is simply that to count someone as thinking about the physical world is *ipso facto* to take her judgments to be *evaluable* by reference to the laws of physics. Someone whose judgments were not so evaluable could still be counted as *thinking*, but not as thinking about the physical world. It is in this sense that Frege holds that a law of physics "can be conceived as prescribing that one ought to think in conformity with it, and is thus in that sense a law of thought" (1893, xv).

On Frege's view, then, laws of physics cannot be distinguished from laws of logic on the grounds that the former are descriptive and the latter prescriptive. Both kinds of laws are descriptive in content but have prescriptive consequences. They differ only in the activities for which they provide constitutive norms. While physical laws provide constitutive norms for thought *about the physical world*, logical laws provide constitutive norms for thought *as such*. To count an activity as thinking about the physical world is to hold it assessable in light of the laws of physics; to count an activity as thinking at all is to hold it assessable in light of the laws of logic. Thus, the kind of generality that distinguishes logic from the special sciences is a generality in the applicability of the *norms* it provides. Logical laws are more general than laws of the special sciences because they "prescribe universally the way in which one ought to think *if one is to think at all*" (1893, xv, my emphasis), as

[19] If by "the laws of physics" Frege means the *true* laws of physics, then the variety of correctness at issue will be *truth*. On the other hand, if by "the laws of physics" he means the laws we currently *take* to be true, then the variety of correctness at issue will be some kind of epistemic *justification*. Either way, the descriptive laws will have normative consequences for our thinking.

opposed to the way in which one ought to think in some particular domain (cf. 1979, 145–46). I'll call this sense of generality "Generality."

Generality and logical objects

We can now answer Hodes's question: how can logic be "topic-neutral" and yet have its own objects? For the kind of generality or topic-neutrality Frege ascribes to logic—normativity for thought as such—does not imply indifference to the distinguishing features of objects or freedom from ontological commitment. There is no contradiction in holding that a discipline that has its own special objects (extensions, numbers) is nonetheless normative for thought as such.

Indeed, Frege argues that arithmetic is just such a discipline. In the *Grundlagen,* he observes that although one can *imagine* a world in which physical laws are violated ("where the drowning haul themselves up out of swamps by their own topknots"), and one can coherently *think about* (if not imagine) a world in which the laws of Euclidean geometry do not hold, one cannot even coherently *think about* a world in which the laws of arithmetic fail:

> Here, we have only to try denying any one of them, and complete confusion ensues. Even to think at all seems no longer possible. The basis of arithmetic lies deeper, it seems than that of any of the empirical sciences, and even than that of geometry. The truths of arithmetic govern all that is numerable. This is the widest domain of all; for to it belongs not only the actual, not only the intuitable, but *everything thinkable. Should not the laws of number, then, be connected very intimately with the laws of thought?* (1884, §14, emphasis added)

Frege's point here is not that it is impossible to judge an arithmetical falsehood to be true—certainly one might make a mistake in arithmetic, and one might even be mistaken about a basic law—but rather that the laws of arithmetic, like the laws of logic, provide norms for thought as such. The contrasts with physics and geometry are meant to illustrate this. The laws of physics yield norms for our thinking insofar as it is about the actual world. The laws of geometry yield norms for our thinking insofar as it is about what is intuitable. But there is no comparable way to complete the sentence when we come to arithmetic. The natural thing to say is that the laws of arithmetic yield norms for our thinking insofar as it is about what is numerable. But this turns out to be no restriction at all, since (on Frege's view) the numerable is just the thinkable. It amounts to saying that the laws of arithmetic yield norms for our thinking insofar as it is ... *thinking*! Hence there is no restricted domain X such that arithmetic provides norms for thinking insofar as it is about X. Whereas in doing non-Euclidean geometry we can say, "we are no longer thinking correctly *about space*, but at least our thought cannot be faulted *qua* thought," it would never be appropriate to say, "we are no longer thinking correctly *about numbers*, but at least our thought cannot be faulted *qua* thought." A judgment that was not subject to the norms of correct arithmetical thinking could not count as a judgment at all.[20]

To see how "complete confusion ensues" when one tries to think without being governed by the norms provided by basic laws of arithmetic, suppose one judges that $1 = 0$. Then one can derive any claim of the form "there are *F*s" by *reductio ad absurdum*. For suppose there are no *F*s. Then, by the usual principles governing the application of arithmetic, the number of *F*s $= 0$.[21] Since $1 = 0$, it follows that the number of *F*s $= 1$, which in turn implies that there are *F*s, contradicting the hypothesis. By *reductio*, then, there are *F*s. In particular (since '*F*' is schematic), there are circles that are not circles. But this is a contradiction. Thus, if we contradict a basic truth of arithmetic like '$1 \neq 0$', we will be committed to contradictions in areas that have nothing to do with arithmetic. Our standards for reasoning will have become incoherent. (Contrast what happens when we deny a geometrical axiom, according to Frege: we are led to conflicts with spatial intuition and experience, but not to any real contradictions.)

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[20] For other passages motivating logicism through arithmetic's normative applicability to whatever is thinkable, see 1885, 94–95, and Frege's letter to Anton Marty of August 29, 1882 (1980, 100). Dummett claims that we must distinguish two dimensions in Frege's talk of "range of applicability"—(i) the generality of the vocabulary used to express a proposition and (ii) the proposition's modal force (that is, its normative generality of application)—and that Frege is concerned with sense (ii) in the 1884 passage and sense (i) in the 1885 passage (1991, 43–44). But as far as I can see, Frege is *nowhere* concerned with generality in sense (i). Unlike Russell, he does not attempt to delineate the logical by reference to features of logical vocabulary. Only once does he raise the question of how logical notions are to be distinguished from nonlogical ones (1906, 429); he never takes it up again (see Ricketts 1997). Moreover, Dummett's reading commits him to finding a *descriptive* (or, in Dummett's terms, nonmodal) reading of Frege's claim that the basic laws of arithmetic "*cannot apply merely to a limited area*" (1885, 95). I have already explained why I am skeptical that this can be done.

[21] Note that we could block this move by divorcing arithmetic from its applications and adopting a kind of formalism about arithmetic. Thus, Frege's argument that arithmetic provides norms for thought as such presupposes his criticisms of formalism (cf. 1903, §§86–103, §§124–137; 1906). Arithmetic as the formalists construe it provides only norms for making marks on paper.

Of course, Frege did not view the argument of §14 as a conclusive proof of the logical or analytic character of arithmetic. (If he had, he could have avoided a lot of hard work!) He insisted that a rigorous proof of logicism would have to take the form of a derivation of the fundamental laws of arithmetic (or their definitional equivalents), using only logical inference rules, from a small set of primitive logical laws (§90).[22] But when it comes to the question what makes a primitive law *logical*, Frege has nothing to say beyond the appeal to Generality in §14. To ask whether a primitive law is logical or nonlogical is simply to ask whether the norms it provides apply to thought *as such* or only to thought in a particular domain. Nothing, then, rules out a primitive logical law that implies the existence of objects (like Frege's own Basic Law V), provided that truths about those objects have normative consequences for thinking as such, no matter what the subject matter.

Generality and Hume's Principle

If the foregoing account of Frege's concept of logic is right, then it answers the question that puzzled Boolos and Hodes: how could Frege have coherently thought that arithmetic, which implies the existence of infinitely many objects, is nothing more than logic? But it raises a question of its own. Nothing in Frege's concept of logic, as I have explicated it, rules out taking "Hume's Principle,"

$$\text{(HP)} \quad (\forall F)(\forall G)(\#F = \#G \equiv F \approx G)$$

as a primitive logical law. (Here '#' is a primitive second-order functor meaning *the number of*, and '$F \approx G$' abbreviates a formula of pure second-order logic with identity that says that there is a one-one mapping from the *F*s onto the *G*s.)[23] For although (HP) is not a *traditional* law of

[22] See also Frege 1897, 362–63. But compare Frege's claim in 1885 that in view of the evident Generality of arithmetic, we "have *no choice* but to acknowledge the purely logical nature of arithmetical modes of inference" (96, emphasis added).

[23] Formally, $F \approx G =_{def} \exists R\, [\forall w(Fw \supset \exists !v\, (Gv \,\&\, Rwv)) \,\&\, \forall w(Gw \supset \exists !v(Fv \,\&\, Rvw))]$.

logic, and *the number of* is not a traditional logical notion,[24] (HP)'s claim to *Generality* seems just as strong as that of Frege's Basic Law V,

$$\text{(BL5)}\quad (\forall F)(\forall G)(\varepsilon F = \varepsilon G \equiv \forall x(Fx \equiv Gx))$$

(where 'ε' is a primitive second-order functor meaning *the extension of*).[25] After all, every concept that has an extension also has a number, so wherever (BL5) is applicable, so is (HP). Of course, in the *Grundlagen* and the *Grundgesetze*, Frege would have had good reason for denying that (HP) is *primitive*: he thought he could define '#' in terms of 'ε' in such a way that (HP) could be derived from (BL5) and other logical laws. But he no longer had this reason after Russell's Paradox forced him to abandon the theory of extensions based on (BL5). Moreover, he knew that all of the basic theorems of arithmetic could be derived directly from (HP), without any appeal to extensions.[26] Why, then, didn't he simply replace (BL5) with (HP) and proclaim logicism vindicated? The fact that he did not do this, but instead abandoned logicism, suggests that he did not take (HP) to be even a *candidate* logical law.[27] And that casts doubt on my contention that Generality is Frege's sole criterion for logicality.

In fact, however, Frege's reasons for not setting up (HP) as a basic logical law do not seem to have been worries about (HP)'s *logicality*. In a letter to Russell dated July 28, 1902—a month and a half after Russell pointed out the inconsistency in (BL5)—Frege asks whether there might be another way of apprehending numbers than as the extensions of concepts (or more generally, as the courses-of-values of functions). He considers the possibility that we apprehend numbers through a principle like (HP), but rejects the proposal on the grounds that "the difficulties here are the same as in transforming the generality

[24] At any rate, not a notion *firmly entrenched* in the logical tradition. Boole wrote a paper (published posthumously in 1868) on "numerically definite propositions" in which "Nx"—interpreted as "the number of individuals contained in the class x"—is a primitive term. In a sketch of a logic of probabilities, he argues that "the idea of Number is not solely confined to Arithmetic, but ... it is an element which may properly be combined with the elements of every system of language which can be employed for the purposes of general reasoning, whatsoever may be the nature of the subject" (1952, 166).

[25] This is a slight simplification: Frege's actual Basic Law V defines the more general notion *the course-of-values of*, but the differences are irrelevant to our present concerns.

[26] See Wright 1983, Boolos 1987, Heck 1993.

[27] See Heck 1993, 286–87.

of an identity into an identity of courses-of-values" (1980b, 141)[28]—which is just what (BL5) does. What is significant for our purposes is that Frege does *not* reject the proposal on the grounds that '#' is not of the right character to be a logical primitive, or (HP) to be a logical law. Indeed, he seems to concede that (HP) is no *worse* off than (BL5) as a foundation for our semantic and epistemic grip on logical objects. The problem, he thinks, is that it is no *better* off, either: the difficulties, he says, are *the same. Neither* principle will do the trick.

Frege's thinking here is liable to strike us as odd. For *we* see the problem with (BL5) as its inconsistency, and (HP) is provably consistent (more accurately, it is provably equiconsistent with analysis (Boolos 1987, 196)). So from our point of view, the difficulties with (HP) can hardly be "the same" as the difficulties with (BL5). But *Frege* didn't have any grounds for thinking that (HP) was consistent, beyond the fact that it had not yet been shown inconsistent. What Russell's letter had shown him was that his methods for arguing (in 1893, §30–31) that every term of the form "the extension of *F*" had a referent were fallacious. He had no reason to be confident that the same methods would fare any better with (HP) in place of (BL5) and "the number of *F*s" in place of "the extension of *F*." Thus the real issue, in the wake of Russell's paradox, was not the *logicality* of (HP), but the *referentiality* of its terms (and hence its truth). It was doubts about this, and not worries about whether (HP), *if true*, would be logical in character, that kept Frege from taking (HP) as a foundation for his logicism.[29]

Given that Frege had grounds for doubt about the *truth* of (HP), then, we need not suppose that he had special doubts about its *logicality* in order to explain why he didn't set it up as a primitive logical law when Russell's paradox forced him to abandon extensions. It is consis-

[28] I have modified the translation in Frege 1980b in two respects: (1) I have used "courses-of-values" in place of "ranges of values," for reasons of terminological consistency, and (2) I have removed the spurious "not" before "the same." The German (in Frege 1980a) is "Die Schwierigkeiten sind hierbei aber dieselben. ..." (I am thankful to Danielle Macbeth and Michael Kremer for pointing out this mistake in the translation.)

[29] When Frege finally gave up on logicism late in his life, it was because he came to doubt that number terms should be analyzed as singular referring expressions, as their surface syntax and inferential behavior suggests. In a diary entry dated March 1924, he writes: "when one has been occupied with these questions for a long time one comes to suspect that our way of using language is misleading, that number-words are not proper names of objects at all and words like 'number', 'square number' and the rest are not concept-words; and that consequently a sentence like 'Four is a square number' simply does not express that an object is subsumed under a concept and so just cannot be construed like the sentence 'Sirius is a fixed star.' But how then is it to be construed?" (1979, 263; cf. 257).

tent with the evidence to suppose that Frege took (HP) and (BL5) as on a par with respect to logicality, as the demarcation of the logical by Generality would require.

Kant's Characterization of Logic as General

It remains to be shown that Kant thinks of logic as General in the same sense as Frege. We have already cleared away one potential obstacle. While Frege conceives of logic as a body of *truths*, Kant conceives of it as a body of *rules*. If we were still trying to understand the sense in which Frege takes logic to be general in *descriptive* terms—for example, in terms of the fact that laws of logic quantify over all objects and all functions—then there could be no analogous notion of generality in Kant. But as we have seen, although Frege takes logic to be a body of truths, he takes these truths to *imply* norms, and his characterization of logic as General appeals only to this *normative* dimension. In fact, his distinction between logical laws, "which prescribe universally the way in which one ought to think if one is to think at all" (1893, xv), and laws of the special sciences, which can be conceived as "prescriptions to which our judgements must conform *in a different domain* if they are to remain in agreement with the truth" (1979, 145–46, emphasis added), precisely echoes Kant's own distinction in the first *Critique* between general and special laws of the understanding. The former, Kant says, are "the absolutely necessary rules of thinking, without which no use of the understanding takes place," while the latter are "the rules for correctly thinking about a certain kind of objects" (*KrV,* A52/B76). The same distinction appears in the *Jäsche Logic* as the distinction between necessary and contingent rules of the understanding:

> The former are those without which no use of the understanding would be possible at all, the latter those without which a certain determinate use of the understanding would not occur. ... Thus there is, for example, a use of the understanding in mathematics, in metaphysics, morals, etc. The rules of this particular, determinate use of the understanding in the sciences mentioned are contingent, because it is contingent whether I think of this or that object, to which these particular rules relate. (*JL*, 12)

The necessary rules are "necessary," not in the sense that we cannot think contrary to them, but in the sense that they are *unconditionally binding* norms for thought—norms, that is, for thought *as such.* (Compare the sense in which Kant calls the categorical imperative "necessary.") Similarly, the contingent rules of the understanding provided by geometry or physics are "contingent," not in the sense that they could have been otherwise, but in the sense that they are binding on our thought only *conditionally*: they bind us only to the extent that we think about space, matter, or energy. (Compare the sense in which Kant calls hypothetical imperatives "contingent.") In characterizing logic as the study of laws unconditionally binding on thought as such, then, Frege is characterizing it in precisely the same way Kant did. Very likely this is no accident: we know that Frege read Kant and thought about his project in Kantian terms.[30]

We are not yet entitled to conclude, however, that Frege's case for the logicality of his system rests on a characterization of logic that Kant could accept. For although we have established that Generality is a *part* of Kant's characterization of logic, we have not yet shown that it is the *whole*. Perhaps Kant could have acknowledged the Generality of Frege's Begriffsschrift—the fact that it provides norms for thought as such—while rejecting its claim to be a *logic*, on the grounds that it is not Formal. In the next section, I will remove this worry by arguing that Formality is for Kant merely a *consequence* of logic's Generality, not an independent defining feature. If Kant could have been persuaded that Frege's Begriffsschrift was really *General*, he would have accepted it as a logic, existential assumptions and all.

3. Formality

Our reading of Kant is likely to be blurred if we assume that in characterizing (general) logic as Formal, he is simply repeating a traditional characterization of the subject. For although this characterization *became* traditional (largely due to Kant's own influence), it was not part of the tradition to which *Kant* was reacting.[31] It is entirely absent, for instance, from the set text Kant used in his logic lectures: Georg Friedrich Meier's *Auszug aus der Vernunftlehre*.[32] Kant's claim that logic

[30] Kitcher 1979, Sluga 1980, and Weiner 1990 have emphasized the extent to which Frege's epistemological project is embedded in a Kantian framework. For evidence that Kant was familiar with the *Jäsche Logik*, see Frege 1884, §12.

[31] For a fuller discussion, see chapter 4 of MacFarlane 2000. It should go without saying that the fact that some pre-Kantian writers use the *word* 'formal' in connection with logic does not show that they think of logic, or a part of logic, as Formal in Kant's sense.

[32] Meier defines logic as "a science that treats the rules of learned cognition and learned discourse" (§1), dividing this science in various ways, but never into a part whose concern is the *form* of thought. Although Meier follows tradition (e.g., Arnauld and Nicole 1662, 218) in distinguishing between material and formal incorrectness in inferences (§360, cf. §§359, 395), the distinction he draws between formal and material is simply skew to Kant's. In Meier's sense, material correctness amounts to nothing more

is purely Formal—that it abstracts entirely from the objective content of thought—is in fact a radical innovation.[33] It is bound up, both historically and conceptually, with Kant's rejection of the "dogmatic metaphysics" of the neo-Leibnizians (among them Meier), who held that one could obtain knowledge of the most general features of reality through logical analysis of concepts.

The neo-Leibnizians agree with Kant about the *Generality* of logic: logic "treats of rules, by which the intellect is directed in the cognition of every being ... : the definition does not restrict it to a certain kind of being" (Wolff 1728, Discursus praeliminaris, §89). But they disagree about its Formality. On the neo-Leibnizian view, the Generality of logic does not require that it abstract entirely from the content of thought. It must abstract from all *particular* content—otherwise it would lose its absolutely general applicability—but not from the most general or abstract content. Thus, although logic abstracts from the contents of concepts like *cat* and *red*, it does not abstract from the contents of highly general and abstract concepts like *being*, *unity*, *relation*, *genus*, *species*, *accident*, and *possible*. Indeed, logical norms depend on general truths about reality that can only be stated using these concepts. For example, syllogistic inference depends on the *dictum de omni et nullo*—"the determinations of a higher being [in a genus-species hierarchy] are in a being lower than it" (Baumgarten 1757, §154)—which the neo-Leibnizians regard as a straightforward truth about reality. And the section of Baumgarten's *Metaphysica* devoted to ontology begins with statements of the principles of non-contradiction, excluded middle, and identity, phrased not as principles of thought but as claims about *things*: "nothing is and is not" (§7); "everything possible is either A or not A" (§10); "whatever is, is that thing" (§11). Logic is still distin-

than the truth of the premises, while formal correctness concerns the *connection* between premises and conclusion. But for Kant, to say that general logic is Formal is not to say that it is concerned with relations of consequence (as opposed to the truth of premises): special logics are also concerned with relations of consequence, and they are not Formal.

[33] The Kantian origin of the doctrine was widely acknowledged in the nineteenth century (De Morgan 1858, 76; Mansel 1851, ii, iv; Trendelenburg 1870, 15). When Bolzano (1837) examines the idea that logic concerns the form of judgments, not their matter—a doctrine, he says, of "the more recent logic"—almost all of the explanations he considers are from Kant (whom he places first) or his followers. British logic books are wholly innocent of the doctrine until 1833, when Sir William Hamilton introduces it in an influential article in the *Edinburgh Review* (Trendelenburg 1870, 15 n. 2). After that, it becomes ubiquitous, and its Kantian origins are largely forgotten. (The story is told in more detail in section 4.5 of MacFarlane 2000.)

guished from metaphysics in being concerned with rules for *thinking*, but (as Wolff puts it) "these should be derived from the cognition of being in general, which is taken from ontology. ... It is plain, therefore, that principles should be sought from ontology for the demonstrations of the rules of logic" (§89). Since thought is about reality, the most general norms for thought must depend on the most general truths about reality.

This is the view to which Kant is reacting when he insists that general logic "abstracts from all contents of the cognition of the understanding and of the difference of its objects, and has to do with nothing but the mere form of thinking" (*KrV*, A54/B78). Our eyes tend to pass without much friction over the words I have just quoted: the idea that logic is distinctively *formal* (in one sense or another) is one to which we have become accustomed. But at the time Kant wrote these words, they would have been heard not as traditional platitudes, but as an explicit challenge to the orthodox view of logic.

Some Relevant Texts

The fact that Kant's claim that logic is Formal is novel and controversial does not, by itself, show that he regards it as a substantive thesis. We might still suppose that he is attempting a kind of persuasive redefinition. However, there are passages in which Kant seems to *infer* the Formality of logic from its Generality. These texts suggest that he regards Formality as a *consequence* of Generality, not an independent defining feature of logic.

For example, consider Kant's discussion of general logic in the *Jäsche Logic*:

> [1] If now we put aside all cognition that we have to borrow from *objects* and merely reflect on the use just of the understanding, we discover those of its rules which are necessary without qualification, for every purpose and without regard to any particular objects of thought, because without them we would not think at all. [2] Thus we can have insight into these rules *a priori, i.e., independent of all experience*, because they contain merely the conditions for the use of the understanding in general, *without distinction among its objects*, be that use *pure* or *empirical*. [3] **And from this it follows** at the same time that the universal and necessary rules of thought in general can concern merely its *form* and not in any way its *matter*. [4] Accordingly, the science that contains these universal and necessary rules is merely a science of the form of our cognition through the understanding, or of thought. (*JL*, 12, boldface emphasis added)

In [1], Kant is adverting to the *Generality* of logical laws: their normativity for thought as such. In [2] and [3], he draws two further conclusions from the Generality of logical laws: they must be knowable a priori and they must be purely *Formal*.[34] [4] sums up: a general logic must also be Formal.

Similar inferences can be found in the *Reflexionen*:

> So a universal doctrine of the understanding presents only the necessary rules of thought irrespective of its objects (i.e., the matter that is thought about), *thus* only the form of thought as such and the rules, without which one cannot think at all. (*R* 1620, at 40.23–25, emphasis added)

> If one speaks of cognition *überhaupt*, then one can be talking of nothing beyond the form. (*R* 2162)

All of these passages seem to conclude that logic is Formal *on the basis of* its Generality. Thus, they support the view that Kant regards the Formality of logic as a *consequence* of its Generality, not an independent defining feature. If this is right, then the disagreement between Kant and the neo-Leibnizians about the Formality of logic is a substantive one, not a dispute over the proper *definition* of 'logic'. Kant and his neo-Leibnizian opponents agree about what logic *is* (the study of norms for thinking as such); they disagree only about what it is *like* (whether or not it abstracts entirely from the contents of concepts, whether it depends in any way on ontology, etc.).

This view receives further support from the fact that Formality plays no essential role in Kant's demarcation of pure general logic from special, applied, or transcendental logics. In the first *Critique*, general logic is distinguished from special logics by its Generality (A52/B76), while *pure* logic is distinguished from applied logic by its abstraction from the empirical conditions of its use (A53/B77). Together, these two criteria are sufficient to demarcate pure general logic; there is no further taxonomic work for an appeal to Formality to do. It is true that, immediately after making these distinctions, Kant describes pure general logic as Formal:

[34] [3] might also be construed as saying that the Formality of logic follows from its a priori knowability. But the interpretation I have suggested seems more natural, especially in view of "at the same time" (*zugleich*), which suggests that [2] and [3] are parallel consequences of [1]. It also makes better sense philosophically. For it does not follow from the a priori knowability of a law that it concerns merely the form of thought "and not in any way its *matter*": if it did, general logic would be the only a priori science. In addition, there are passages in which Kant infers the Formality of logic directly from its Generality, with no mention of a priori knowability (see below).

> A general but pure logic therefore has to do with strictly *a priori* principles, and is a canon of the understanding and reason, but only in regard to what is formal in their use, be the content what it may (empirical or transcendental). (A53/B77)

But this passage is best construed as drawing *consequences* from the taxonomy Kant has just provided (note the 'therefore'), not as providing a further *differentia* of pure general logic.

Although it is sometimes thought that Formality is needed to distinguish general logic from *transcendental* logic, this is not the case. It is easy to be misled by the fact that Kant appeals to Formality in describing the difference between general logic and transcendental logic:

> General logic abstracts, as we have shown, from all content of cognition, i.e. from any relation of it to the object, and considers only the logical form in the relation of cognitions to one another, i.e., the form of thinking in general. But now since there are pure as well as empirical intuitions (as the transcendental aesthetic proved), a distinction between pure and empirical thinking of objects could also well be found. In this case there would be a logic in which one did not abstract from all content of cognition ... (A55/B79–80)

But this appeal to Formality does no independent taxonomic work, for transcendental logic is already sufficiently distinguished from general logic by its lack of Generality. Transcendental logic supplies norms for "the pure thinking *of an object*" (A55/B80, emphasis added), not norms for thought *as such*. Accordingly, it is a *special* logic.[35] Indeed, the way Kant begins the paragraph quoted above—"General logic abstracts, *as we have shown*, from all content of cognition ..."—would be quite odd if he regarded the connection between Formality and general logic as definitional.

All of this evidence suggests that Kant's claim that general logic is Formal is a substantive *thesis*, not an attempt at "persuasive definition." But if so, what are Kant's *grounds* for holding this thesis?

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[35] Kant seems to regard the restriction of transcendental logic to objects capable of being given in human sensibility as a *domain restriction*, like the restriction of geometry to spatial objects. Thus, for instance, he says that transcendental logic represents the object "as an object of the mere understanding," while general logic "deals with all objects in general" (*JL*, 15). And in *R* 1628 (at 44.1–8), Kant uses "objects of experience" as an example of a particular domain of objects that would require special rules (presumably, those of transcendental logic)—as opposed to the "rules of thinking *überhaupt*" contained in general logic. These passages imply that transcendental logic is a special logic, in Kant's sense. Still, I am not aware of any passage in which Kant explicitly *says* this.

From Generality to Formality

Kant nowhere gives an explicit argument for the thesis that general logic must be Formal. However, it is possible to reconstruct such an argument from Kantian premises. The conclusion follows directly from two key lemmas:

(LS) General logic must abstract entirely from the relation of thought to sensibility.

and

(CS) For a concept to have content is for it to be applicable to some possible object of sensible intuition.

Given (CS), it follows that to abstract from the relation of thought to sensibility is to abstract from the contents of concepts. So if general logic must abstract entirely from the relation of thought to sensibility, as (LS) claims, then

(LC) General logic must abstract entirely from the contents of concepts.

In other words, it must be Formal.

It remains to give Kantian arguments for the two lemmas. (LS) is the most straightforward. On Kant's view,

(TS) Thought (thinking) is intelligible independently of its relation to sensibility.

Though Kant holds that cognition of an object requires both thought and sensibility, he holds that the contributions of the two faculties can be distinguished (*KrV*, A52/B76). And not just notionally: Kant insists that

> the categories are not restricted in *thinking* by the conditions of our sensible intuition, but have an unbounded field, and only the *cognition* of objects that we think, the determination of the object, requires intuition; in the absence of the latter, the thought of the object can still have its true and useful consequences for the *use* of the subject's *reason*, which, however, cannot be expounded here, for it is not always directed to the determination of the object, thus to cognition, but rather also to that of the subject and its willing. (B166 n.; cf. Bxxvi)

As Parsons points out, Kant's metaphysics of morals presupposes the possibility of this "problematic" extension of thought beyond the bounds of sense (1983, 117).

The first lemma follows almost immediately from this premise. For as we have seen,

(GL) General logic concerns itself with the norms for thought *as such.*

But since thought is intelligible independently of its use in relation to sensibility (TS), the norms for thought *as such* cannot depend in any way on the relation of thought to sensibility. Thus,

(LS) General logic must abstract entirely from the relation of thought to sensibility.

The argument for the second lemma is more involved. Here we need three premises. First,

(CJ) For a concept to have *content* is for it to be usable in a judgment.

This is an expression of what is sometimes called "the primacy of the propositional."[36] On Kant's view, "the only use which the understanding can make of these concepts is to judge by means of them" (A68/B93). A "concept" that could not be used in any possible judgment would have no objective significance, no semantic content, at all.

Second,

(JO) Judgment is the mediate cognition of an object.

For Kant, what distinguishes a judgment (which is capable of being true or false) from a mere subjective association of representations (which is not) is that in a judgment, the representations are claimed to be "combined *in the object*" (B142). Thus, judgment is *essentially* "the mediate cognition of an object, hence the representation of a representation of it" (A68/B93). The subject concept in every judgment must relate finally to a representation that is "related immediately to the object" (A68/B93)—that is, to a singular representation, or intu-

[36] Cf. Brandom 1994: "One of [Kant's] cardinal innovations is the claim that the fundamental unit of awareness or cognition, the minimum graspable, is the *judgment.* ... for Kant, any discussion of content must start with the contents of judgments, since anything else only has content insofar as it contributes to the contents of judgments" (79–80). Note that Kant's word 'judgment' is broader in its application that ours. A proposition that is merely entertained, or one that forms the antecedent of a conditional, still counts as a judgment for Kant: a "problematic" one (*KrV,* A75/B100; *JL,* §30).

ition.[37] Otherwise, there would be nothing—*no thing(s)*—for the putative judgment to be *about*, and it would not be a cognition at all:

> For two components belong to cognition: first, the concept, through which a object is thought at all ..., and second, the intuition, through which it is given; for if an intuition corresponding to the concept could not be given at all, then it would be a thought as far as its form is concerned, but without any object, and by its means no cognition of anything at all would be possible, since, as far as I would know, nothing would be given nor could be given to which my thought could be applied. (B146)

On Kant's view, then, there can be no such thing as a judgment about concepts themselves: the objective purport of judgment gets spelled out in terms of the *relation of concepts to an object or objects*.[38]

Third,

(OS) Objects can be given to us only in sensibility. That is, the only intuitions (singular representations) we are capable of having are sensible.

"It comes along with our nature," Kant says, "that *intuition* can never be other than *sensible*; i.e., that it contains only the way in which we are affected by objects" (A51/B75; cf. A19/B33, A68/B92, A95, B146, A139/B178). In this we differ from God, whose intuition is "intellectual" or "original" (B72). God has singular representations not through being affected by objects, but through creating them.

In addition to these three premises, we will also need a *logical lemma*:

(SC) If a concept can be used in a judgment at all, then it can be used as the subject concept of a categorical judgment.

Though this lemma is needed for the argument for (CS), and I am inclined to think that Kant would accept it, I know of no text in which he explicitly endorses it. However, it is plausible in light of Kant's log-

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[37] For the definition of 'intuition' as "singular representation," see *JL*, §1. Kant sometimes adds that intuitions relate *immediately* to their objects (*KrV*, A320/B377). I do not think that immediacy and singularity are distinct conditions for Kant: to say that concepts are general is just to say that they relate only mediately to objects (that is, through their marks); if these marks pick out only a single object, that does not make the concept singular. For a fair-minded discussion of this issue with references to the literature, see Parsons 1983, 111–14, 142–49.

[38] *Analytic* judgments are no exception. Although we need not look beyond the concepts themselves to know the truth of an analytic judgment and can therefore abstract from their relation to objects (A258/B314), analytic judgments are still judgments about *objects*, not concepts (cf. Paton 1936, 214 n. 3). Without "relation to an object" they would not be judgments at all.

ical views. First, notice that if a concept is used in a hypothetical or disjunctive judgment, then it is used in a categorical judgment, for hypothetical and disjunctive judgments are made up of categorical judgments (*JL*, §§25, 28; *KrV*, A73/B98–99). So it suffices to show that if a concept can be used as the predicate concept of a categorical judgment, then it can also be used as the subject concept of a categorical judgment. In other words, it suffices to rule out the possibility of a concept that could be used only as the predicate concept of judgments, and never as the subject concept. It is easy to rule out this possibility for judgments of universal affirmative, particular affirmative, and universal negative form. For Kant accepts the Aristotelian "conversion" inferences, in which subject and predicate switch places (*JL*, §§51–53):

All *A* are *B*; therefore, some *B* are *A*.
Some *A* are *B*; therefore, some *B* are *A*.
No *A* are *B*; therefore, no *B* are *A*.

If a concept were usable as the predicate of a categorical judgment of one of these forms, but not as the subject of any categorical judgment, then a logically valid inference would take us from a judgment to a non-judgment—surely not something Kant wants to allow. The only remaining case is that of particular negative judgments, of the form "Some *A* are not *B*." Might there be concepts that could be used as predicates in judgments of this form, but never as subjects in any judgment? Presumably not. If "some *A* are not *B*" can be a judgment, then presumably "some *A* are *B*" can also be a judgment. But then "some *B* are *A*" can be a judgment, so *B* can be used as the subject concept of a judgment after all.

We can now prove (CS). From (CJ), (JO), and (SC), it follows that

(CO) For a concept to have content is for it to be applicable to some object that could be given in an intuition (singular representation).

For a concept to have content is for it to be usable in some possible judgment (CJ), and hence (SC) as the subject concept of some possible categorical judgment. But judgment is essentially the mediate cognition of an object (JO): the subject concept in a categorical judgment must apply to some object that could be given in an intuition. Thus,

> For every concept there is requisite, first, the logical form of a concept (of thinking) in general, and then, second, the possibility of giving it an object to which it is to be related. Without this latter it has no sense, and

> is entirely empty of content, even though it may still contain the logical function for making a concept out of whatever sort of *data* there are. (A239/B298; cf. A69/B93–94, B147, B148–49, A139/B178, A146/B185, A147/B186, A242/B300, A246/B302, A247/B304)

For example, the concept of *body* is a concept "only because other representations are contained under it by means of which it can be related to objects" (A69/B94).[39]

Finally, from (CO) and (OS), it follows trivially that

> (CS) For a concept to have content is for it to be applicable to some possible object of sensible intuition.

Thus, thought has content only through its relation to sensibility:

> the condition of the objective use of all our concepts of understanding is merely the manner of our sensible intuition, through which objects are given to us, and, if we abstract from the latter, then the former have no relation at all to any sort of object. (A286/B342)

As we have seen, this lemma, together with (LS), is sufficient to underwrite Kant's thesis that logic, if it is to be General, must also be Formal.

Some Historical Confirmation

I have given some textual evidence that Kant *infers* the Formality of logic from its Generality, and I have shown that he could have based such an inference on his substantive views about thought, judgment, concepts, and intuitions. But to say he *could have* is not to say that he did. Is there any reason to believe that the premises I have isolated above were Kant's actual grounds for thinking that general logic is Formal?

Yes. A key lemma of the argument, as I have reconstructed it, is that thought has content only through its relation to sensibility. But this claim is distinctive of Kant's mature critical philosophy: he did not hold it prior to the period in which he was developing his critical view, 1772–75.[40] So if this argument gives Kant's reasons for thinking that logic is Formal, we should expect talk of the Formality of logic to be absent

[39] Note that (CO) holds of mathematical concepts as well as empirical ones: mathematical concepts give one a priori cognition of objects, but only as regards their *forms* (as appearances); their content is contingent on the supposition "that there are things that can be presented to us only in accordance with the form of that pure sensible intuition" (B147, cf. A239–40/B298–99). See Thompson 1972, 339–42.

[40] For the justification of this date range, see de Vleeschauwer 1939, 65–66, Guyer and Wood in Kant 1998, 46–60, and pages 55–56 below.

from his writings before that period. In particular, we should expect such talk to be absent from the transcripts of his logic lectures dated before 1772–75 (the Blomberg and Phillipi Logics) and present in those dated later (the Vienna, Pölitz, Busolt, Dohna-Wundlacken, and Jäsche Logics).[41] And this is precisely what we find. The later logic lectures all characterize logic as concerned with the form of thought, abstracting from content (*DWL*, 693–94; *VL*, 791; *JL*, 12; *BuL*, 609; *PzL*, 503). These claims are absent from the corresponding sections of the earlier lectures, which instead follow Meier's characterization of logic closely.[42] Moreover, although the earlier lectures do characterize logic as *General* (e.g., *PhL*, 314), they contain many claims that are incompatible with the *Formality* of logic. For example, in the Blomberg Logic, Kant echoes Wolff in making logic epistemologically posterior to ontology: "Our rules have to be governed by those universal basic truths of human cognition that are dealt with by *ontologia*. These basic truths are the *principia* of all sciences, consequently of logic too" (28). And in the Phillipi Logic, Kant calls logic an "organon of the sciences" (5), contradicting his later view that logic, because it "abstract[s] wholly from all objects," cannot be an organon of the sciences (*JL*, 13).

It appears, then, that Kant's characterization of logic as Formal dates from the period in which he was writing the first *Critique*. To the extent that we can trust Adickes's dating of the marginalia from Kant's text of Meier, they support this view.[43] The earliest notes characterize logic in much the same way as the early lectures. The first hint that logic must

[41] The earlier lectures date from the early 1770s, while the later ones date from 1780 to 1800. Translations of the Blomberg, Vienna, Dohna-Wundlacken, and Jäsche Logics can be found in Kant 1992a; translations from the other lectures and from Kant's *Reflexionen* are my own.

[42] The Blomberg Logic does distinguish between the formal and the material in cognition (that is, between the manner of representation and the object (*BL*, 40; cf. *PhL*, 341)). Here Kant goes beyond the passage of Meier on which he is commenting (§11–12), which merely distinguishes the cognition from its object. But Kant goes on to say: "Logic has to do *for the most part* with the formal in cognition" (my emphasis), employing a qualification that can have no place in his later view of logic.

[43] These marginalia are collected in Kant *Ak* 16. For Adickes's methodology in assigning dates to the passages, see his introduction to *Ak* 16, esp. xxxv–xlvii. Relevant *Reflexionen* include 1579, 1603, 1608, 1612, 1620, 1624, 1627, 1629, 1721, 1904, 2142, 2152, 2155, 2162, 2174, 2178, 2225, 2235, 2324, 2834, 2851, 2859, 2865, 2871, 2908, 2909, 2973, 3035, 3039, 3040, 3045, 3046, 3047, 3053, 3063, 3070, 3126, 3127, 3169, 3210, 3286. Also relevant are *Reflexionen* 3946 and 3949 from Kant *Ak* 17 (Kant's marginalia on Baumgarten's *Metaphysica*).

be Formal if it is to be General occurs in the midst of a long *Reflexion* Adickes dates from the early 1760s to the mid-1770s:

> Logic as canon (analytic) or organon (dialectic); the latter can not be dealt with universally [=Generally], because it is a doctrine of the understanding not according to the form, but rather according to the content. (*R* 1579)

This *Reflexion* contains at least two temporal strata of comments, and the passage quoted is marked by Adickes as a later interpolation. If we put it towards the end of Adickes's date range, it dates from 1773–75, which is just what we'd expect. There are only three other passages dated before 1773–75 that assert the Formality of logic (*R* 1721, 3035, 2865), and in each case, Adickes expresses uncertainty about the dating and gives 1773–75 as an alternative. On the other hand, there are many passages dated 1775 and later that characterize logic as Formal (e.g., *R* 2155, 2162, 4676). Thus, the *Reflexionen* corroborate what we find in the logic lectures: that Kant's insistence on the Formality of general logic dates roughly from the beginning of his elaboration of his mature critical philosophy in 1773–75.[44]

These historical facts would be puzzling indeed if in claiming that logic is Formal, Kant were merely repeating a traditional characterization of the subject. But they are explained admirably by our reconstruction of Kant's grounds for this claim. In the dissertation of 1770, Kant endorses (TS) and (OS),[45] but he makes claims that are incompatible with (CO). Although he holds that the objects of the senses are "things *as they appear*," he also claims that concepts can relate directly to "things *as they are*," of which we can have no sensuous intuition (*ID*, §4), and hence no singular representation at all. But he gives no account of *how*

[44] This is not to deny that, as Allison (1973, 54) and Longuenesse (1998, 150 n. 26) have emphasized, there are anticipations of this view in Kant's earlier, precritical works (e.g., *ID*, 393; *D*, 295; *B*, 77–78). These passages show movement away from an ontologized Wolffian conception of logic and towards Kant's mature conception of logic as concerned with the form of thought in abstraction from all content. But they are stages along the way, not the finished product. A rationalist might distinguish between "formal" and "material" principles, taking them to be principles of both thought *and* being, but it is essential to Kant's mature view that the forms of thought not be confused with the forms of being. In adopting Crusius's distinction between formal and material principles and limiting logic to the former, the precritical Kant has taken a first step towards a de-ontologized logic. But he still has not taken the decisive second step: declaring that logic abstracts entirely from relation to the content of thought. Kant does not make that claim until he has abandoned the idea that knowledge is possible through concepts alone, without relation to sensibility.

[45] For (TS), see *ID*, §3; for (OS), §10.

concepts can relate to objects of which we can have no intuitions. It is Kant's growing despair at filling this lacuna that leads him down the path to transcendental idealism. In 1772 he writes to Herz:

> In my dissertation I was content to explain the nature of intellectual representations in a merely negative way, namely, to state that they were not modifications of the soul brought about by the object. However, I silently passed over the further question of how a representation that refers to an object without being in any way affected by it can be possible. I had said: The sensuous representations present things as they appear, the intellectual representations present them as they are. But by what means are these things given to us, if not by the way in which they affect us? And if such intellectual representations depend on our inner activity, whence comes the agreement that they are supposed to have with objects—objects that are nevertheless not possibly produced thereby? (February 21, 1772; *Ak* 10:129–35, trans. 1967, 72)

By 1775, Kant has resolved the difficulty by accepting (CO); he now explains the objectivity of pure concepts of the understanding through their applicability to the objects of *empirical* intuitions (as principles of order): "We have no intuitions except through the senses; thus no other concepts can inhabit the understanding except those which pertain to the disposition and order among these intuitions" (*R* 4673, trans. Guyer and Wood in Kant 1998, 50). Now (LC) is inescapable, and Kant soon starts characterizing logic as "formal" (e.g., at *R* 4676).

In calling logic Formal, then, Kant is not giving a persuasive redefinition, but drawing a conclusion from substantive philosophical premises and a neutral, accepted characterization of logic as General. What makes this hard to see, from our perspective, is that because of the enormous influence of Kant's writings on nineteenth-century work in the philosophy of logic, Formality came to be seen as a *defining* characteristic of logic, even by philosophers who rejected Kant's general philosophical outlook. As Trendelenburg observes,

> It is in Kant's critical philosophy, in which the distinction of matter and form is thoroughly grasped, that *formal* logic is first sharply separated out; and properly speaking, it stands and falls with Kant. However, many who otherwise abandon Kant have, at least on the whole, retained formal logic. (1870, 15, my translation)

One result was a blurring of the distinction between Formality and Generality. Now that we have recovered this distinction and seen how Generality and Formality are related in Kant, let us return to the question with which we started: how can Frege avoid the charge that in claiming his Begriffsschrift as a logic, he is simply "changing the subject"?

4. Kant and Frege

The worry was that a "non-Formal logic" would be, for Kant, a *contradictio in adjecto.* We can now put this worry to rest. We have seen that Kant and Frege *agree* that the fundamental defining characteristic of logic is its *Generality*: the fact that it provides norms for thought *as such.* And although Kant holds that a General logic must also be Formal, we have seen that he regards this as a substantive thesis of his critical philosophy, not a matter of definition or conceptual analysis. Thus, Frege can reject the connection between Generality and Formality without "changing the subject," provided he rejects at least one of the premises on which Kant's thesis rests.

In fact, he rejects two of them: (JO) and (OS). His grounds for rejecting both are rehearsed in the first part of the *Grundlagen.* In claiming that ascriptions of number are assertions *about concepts,* I will show, Frege is rejecting (JO), while in insisting that numbers are objects, he is rejecting (OS).

Frege's Rejection of (JO)

Frege claims that ascriptions of number, like 'Venus has 0 moons' or 'the King's carriage is drawn by four horses,' are about *concepts* (here, *moon of Venus, horse drawing the King's carriage*), and not the objects being numbered (the moons, the horses) (1884, §46). Indeed, on Frege's view, even ordinary categorical claims like 'all whales are mammals' are assertions about concepts (here, *whale* and *mammal*), not about any object or objects (§47).[46]

In making these claims, Frege is rejecting Kant's view of judgment—that is, propositional thought—as the mediate cognition of an object. On Kant's view, there is no such thing as a judgment *about concepts.* We use concepts to make claims about objects; where there is no object in which the concepts are claimed to be combined, there is no objective purport, no judgment, no truth or falsity. Frege's claim that certain thoughts are about concepts alone directly contradicts this view. Ascriptions of number, as Frege understands them, do not involve the

[46] Cf. Frege 1895, 454: "If I utter a sentence with the grammatical subject 'all men', I do *not* wish to say something about some Central African chief wholly unknown to me."

subsumption of objects under concepts at all; yet they are clearly objective judgments. (JO), then, must be rejected.

Rejecting (JO) frees Frege to reject (CO), the thesis that for a concept to have content is for it to be applicable to some object of which we could have a singular representation. Frege emphasizes that even self-contradictory concepts (like *rectangular triangle*) have objective content, despite the fact that there could be no object to which they applied, because they can be used in propositions asserting that they have no instances (1884, §53, §74, §94; 1895, 454; 1891, 159; 1894, 326–27; 1979, 124). In fact, Frege defines the number 0 (and indirectly the other numbers as well) in terms of the self-contradictory concept *not identical to itself* (1884, §74). Here his departure from the Kantian view is most striking: for Kant, "The object of a concept which contradicts itself is nothing *because the concept is nothing*, the impossible, like a rectilinear figure with two sides" (*KrV*, A291/B348, emphasis added; cf. A596/B624 n.).

Frege's rejection of (CO) breaks the Kantian chain linking conceptual content with sensibility. Abstraction from sensibility no longer requires abstraction from content, and Kant's inference from the Generality of logic to its Formality is blocked. Frege is entitled to reject this inference, then, because he rejects Kant's way of spelling out the objective purport of thought (and hence of concepts) in terms of its *relation to objects*. I will not attempt to get to the bottom of this dispute here.[47] It should be clear, however, that it is a substantive issue in general philosophy, not a verbal issue about what deserves to be called 'logic'.

Frege's Rejection of (OS)

Let us now turn to (OS). Frege's rejection of (OS) is bound up with his construal of numbers as objects: "I must also protest against the generality of Kant's dictum: without sensibility no object would be given to us. Nought and one are objects which cannot be given to us in sensation" (1884, §89). This reasoning is pretty compelling, even in advance of the logicist reduction, provided one agrees with Frege that the numbers are objects. (OS) is plausible only if one denies, as Kant does, that the numerals refer to objects: on Kant's view, arithmetic applies directly to

[47] What would be needed is a full discussion of Frege's concept/object and sense/reference distinctions. It is Kant's conflation of these two distinctions that forces him to understand the objective purport of judgment in terms of the relation of concepts to objects. Having pulled apart these distinctions, Frege can understand objective purport in terms of the determination of reference by sense, not the relation of concepts to objects. I am indebted here to Danielle Macbeth.

magnitudes given from outside arithmetic, for example, spatial magnitudes.[48] Frege holds, by contrast, that because numerical terms in true arithmetical statements behave grammatically and inferentially like names of objects—for example, they can be formed using definite descriptions and used in genuine identity statements that license intersubstitution (§57), they have no plurals (§68 n.), and they do not function logically like adjectives (§§29–30)—they *are* names of objects (§57).

It doesn't matter for our purposes who is right about this. What matters is that the issue is not primarily one about logic. The reasons Frege gives for thinking that numbers are objects do not presuppose any of his views about logic or the reducibility of arithmetic to logic. In particular, they do not presuppose that numbers are extensions or that extensions are logical objects. Having argued that numbers are objects, Frege faces a problem about how they can be given to us (§62), which he solves by arguing that we grasp numbers as extensions of concepts (§68). But Frege's reasons for thinking that numbers are nonsensible objects are independent of this particular solution to the problem of how they are given to us. Even after his theory of extensions has collapsed, he continues to believe, on the basis of the grammatical and inferential behavior of number words, that numbers are objects. As late as 1924, he is still capable of writing:

> Numerals and number-words are used, like names of objects, as proper names. The sentence 'Five is a prime number' is comparable with the sentence 'Sirius is a fixed star'. In these sentences an object (five, Sirius) is presented as falling under a concept (prime number, fixed star) (a case of an object's being subsumed under a concept). By a number, then, we are to understand an object that cannot be perceived by the senses. (1979, 265)

Evidently, then, Frege's reasons for rejecting (OS) do not depend on a prior commitment to "logical objects" or a prior rejection of the view that logic is Formal.[49]

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[48] See Parsons 1983, 147–49, Friedman 1992, 112–13.

[49] One might wonder, in assessing their disagreement over (OS), whether Kant and Frege mean the same thing by 'object'. Might *this* disagreement be "merely verbal," and if so, haven't I just shifted the bump in the rug from 'logic' to 'object'? I think not. In disputes involving words as centrally embedded in a theoretical framework as 'object', it is usually impossible to make any useful distinctions between semantic and substantial questions. For just this reason, such disputes are *never* "merely verbal." Newton defined momentum as rest mass times velocity, while Einstein rejected this equation; their disagreement, like many interesting scientific and philosophical disputes, was neither entirely factual nor entirely semantic. I would be content to have shown that the issue between Kant and Frege about the Formality of logic depends on disagreements of this kind.

5. Conclusion

We started with an evident difference between Kant's and Frege's *conceptions* of logic: Kant holds that logic is Formal, while Frege denies this. The worry was that in view of this difference, the disagreement between them about the reducibility of arithmetic to "logic" might turn out to be merely verbal. Frege might, as Poincaré, Michael Wolff, and others have charged, have simply *changed the subject.* I hope to have shown that this charge is unfounded. Kant and Frege *agree* in demarcating logic by its Generality; it's just that in the context of Kant's other philosophical commitments, Generality *implies* Formality. Because Frege rejects enough of Kant's general philosophical picture, he can coherently demarcate logic as General in exactly the same sense as Kant, while rejecting Kant's conclusion that it must be Formal. Despite its extravagant ontological commitments, then, Frege's Begriffsschrift *could have been* Logic—in Kant's most narrow and exacting sense—if only it had been consistent.

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