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# UNITY AND INFINITY: PARMENIDES 142b–145a

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AT *Parmenides* 142b–145a, Parmenides attempts to prove that Unity, if it is and thus partakes of Being, is one and many, whole and parts, limited and unlimited in multitude. His argument begins with a proof that Unity, if it is, is a whole whose parts are being and unity (142c–d). This conclusion is then shown to imply that Unity is infinitely divisible, each part containing unity and being as parts (142d–143a). Next, a new principle of division is introduced: Unity, considered in abstraction from its own being, is found to be different from that being (143a–b). This latter Unity is shown to imply the existence of the numbers, which are infinite in multitude (143c–144a). Since Being is distributed to all of the numbers, Being has infinitely many parts (144a–c). But since Unity is co-extensive with Being, it too has infinitely many parts (144c-e). And since Unity is a whole which contains parts, it is limited with respect to its wholeness; therefore, it is part and whole, one and many, limited and unlimited in multitude (145a).

There are a variety of puzzling features about this argument. One of them—questions of validity apart—is its apparent redundancy. Parmenides' initial division provided him with an infinite plurality of parts. He might therefore have given an existence proof of infinitely many numbers, conceived as pluralities of units, by means of this division. Instead, he introduces a new principle of division for the purpose. Again, he derives the conclusion that Unity has infinitely many parts from the infinity of number; but he has already shown that Unity is infinitely divisible into parts consisting of being and unity. In short, the new principle of division introduced for number seems unnecessary, and Parmenides has twice shown that Unity has infinitely many parts.

One way of explaining this redundancy would be to claim that the meaning of "Unity" has shifted: the Greek expression, "one" or "the one," is after all multiply ambiguous. The Unity which is shown to have infinitely many parts consisting of being and unity is a Unity which is, τὸ ἕν ὄν (142d). But the Unity which is shown to have infinitely many parts distributed in correspondence to the num-

bers is a Unity considered apart from its own being (143a). There is, then, a distinction between τὸ ὂν ἕν and αὐτὸ τὸ ἕν, and this is put in stated terms at 144e5–6. The existence of this distinction, however, does not explain why Parmenides should use two arguments to prove that Unity is unlimited in multitude, since it implies no ambiguity in the meaning of "Unity." The distinction rests only on abstraction (τῇ διανοίᾳ . . . λάβωμεν, 143a7), and the force of this is indicated by the fact that to consider Unity apart from its own being implies that it is *different* from its own being (143b), a premise which lays a foundation for the deduction in Hypothesis V[1] that Unity, if it is not, can be spoken of and is different from other things, and thus in some sense possesses being (160c–d, 161e–162b). Parmenides is not dealing with different Unities, but with the same Unity conceived in different ways. It is the same thing which is twice proved to be infinite in multitude. Despite the difference in the way in which Unity is conceived in the two proofs, then, apparent redundancy remains.

This redundancy, however, is merely apparent, not real. I shall argue that Parmenides is dealing, not with one but with two kinds of infinity: a dense infinite allied to that of continuity, typical of extensive magnitudes, and the infinity of succession, typical of number. These two types of infinity were distinct in Greek mathematics in a way in which they are not in our own, since the Greeks, identifying number with the positive integers greater than 1, and thus lacking both reals and rationals, had no concept of number as either continuous or dense, and no notion of isomorphism between numbers and (say) lines. In the *Parmenides,* Unity is proved to have infinitely many dense parts, and infinitely many successive parts, and its simultaneous possession of these different kinds of parts explains the apparent redundancy of the argument. If this account is correct, the *Parmenides* anticipates important features of Aristotle's analysis of infinity and continuity in the *Physics,* though there are also im-

[1] I use Cornford's ordering because it has become conventional to do so, and without prejudice to the issue of the logical structure of the Hypotheses. The assumption that their order is serial is at least as old as Proclus; but this is a substantive issue in interpretation, since that assumption does not answer to the surface structure of the Hypotheses, implies that they are ambiguous, and implies that the conclusion of the Hypotheses at 166c is only ostensible.

portant differences. Still, this is evidence that Aristotle drew on a tradition of physical speculation inherited from the Academy, and that the *Physics* may therefore be looked to as an important source of understanding for given arguments in the *Parmenides*, especially those involving place, time, and motion.[2]

The mathematics of this passage depends on the distinction of part and whole. That distinction, in the *Parmenides*, is complex, and contains metaphysical issues as elements in its complexity. Unity, as a dense infinite, is proved to be a whole of parts through its participation in Being. As a successive infinite, it is proved to be a whole of parts, not through its participation in something else, but through the participation of infinitely many things, the numbers, in it. The issues of participation raised here are important, not only to the *Parmenides*, but to Plato's later metaphysics. 142b–145a is a rich and ample passage. The present discussion of it is meant, not as a last word, but as a next word.

## I

*Paraphrase* (142b–d, on Part and Whole): Unity, since it is and thus partakes of Being, has parts. For if being belongs to the Unity which is (or, to anything which is one, τοῦ ἑνὸς ὄντος), and if unity belongs to the being which is Unity (or, to any one thing which is, τοῦ ὄντος ἑνός), and if being and unity are not the same, while belonging to the same thing,[3] namely Unity, then Unity is a whole, of which its being and unity are parts. Since parts are parts of a whole, Unity (or, anything which is one, ὃ ἂν ἓν ᾖ cf. 129a2) is a whole and has a part.

A salient feature of this argument is its ambiguity: both grammatically and in terms of sense, its subject may be abstract, Unity or the property of being one, or it may be distributively generic, what(ever) is one. This ambiguity is surely intentional: it is essential for the argument which follows that Unity be itself one thing. But that

[2] Here is an example of how useful the *Physics* can be. In Hypothesis I it is argued that Unity, being one and not a whole of parts, cannot be in itself, since if it were, it would contain itself and be contained by itself, in which case it would be, not one, but two. The sense of this dark saying is considerably illuminated by *Physics* IV 210a25ff., where Aristotle concludes that a thing may be said to be in itself when it is said to be so not *qua* itself, but *qua* being a whole of parts; he then goes on to argue that nothing can be in itself in virtue of its own nature. It is a pretty gloss. For other possible examples, see F. M. Cornford, *Plato and Parmenides* (London, 1939), pp. 201–02, and G. E. L. Owen, "*Tithenai ta Phainomena*," in *Aristotle* (ed. Moravcsik) (New York, 1967).

[3] For construction, see F. M. Cornford, *Plato and Parmenides* (London, 1939), p. 137, n.1. Hereafter abbreviated *PP*.

the subject of the argument, "that which we have hypothesized" or τὸ ἕν (142d3), is Unity, is shown by the fact that what has been hypothesized is coordinate with Being (ἡ οὐσία), and distributed to the numbers[4] (144c–d). But if Being is distributed to the numbers (144b1), then each of the numbers partake of Being (144a8). Therefore, if τὸ ἕν is distributed to the numbers, it must be Unity of which the numbers partake. But although Unity is its primary subject, the argument of 142c–d is so constructed as to apply generally to anything which is and is one. Thus it is that τὸ ἓν ὄν at 143a5 is equivalent to τὸ ὂν ἕν at 144e5.

In treating Unity as itself one, the argument recalls Parmenides' Third Man, which had assumed that Largeness is large (131e–132b). Whatever the reason for the latter assumption, a very good reason is given for the former. It is that being and unity are coextensive: whatever is one is, and whatever is is one. Since, by hypothesis, Unity is, Unity is one.

The claim that unity and being are coextensive was assumed in Hypothesis I (141e9–12), and is later restated with greater explicitness (144c–e); it was accepted by Aristotle, who gave as a reason for saying that unity and being are in a sense the same that, "to be one is just to be a particular thing."[5] Aristotle, though he supposed that being and unity were *in a sense* the same, as being coextensive, did not suppose that they were the same. He commented of the historical Parmenides that, "It is necessary for him, then, to assume not only that 'being' has the same meaning of whatever it is predicated, but further that it means (1) what just is and (2) what is just one."[6] But the Platonic Parmenides, like Aristotle, distinguishes: "to be" signifies something different from "to be one."[7]

This bears on the fact that the being and unity of Unity are *parts* of Unity, a claim which must be understood through Parmenides' inference at 142b–c that if Unity partakes of Being, the

[4] The use of initial capitals in "Being" and "Unity" is meant to distinguish them, as distributed, from the being and unity of the things to which distribution is made.

[5] See *Meta.* X 1054a13–19, IV 1003b22–34, VII 1040b16, XI 1061a15–18. Quotations from Aristotle are taken from the Oxford Translation.

[6] *Phys.* I 189a33ff., cf. *Sophist* 244b–c.

[7] 142c4–5, d2–3.

being of Unity is not identical with Unity. There is, then, a distinction between Being and the being *of* something, and this distinction is confirmed at 144b–c in the case of the being of numbers. In a similar way, Socrates had earlier granted a distinction between Likeness itself and the likeness which we possess.[8] It is consistent with this that Cornford should have suggested that the term "part" should be so construed as to include "any and every diversity of aspect or character."[9] This is not false, but unless carefully qualified, it is likely to be misleading. In the first place, the term "part," if it does not have two senses, surely has two uses: at 142b–c, *f* is part of *x* if *x* partakes of F-ness, and this leads to a dense infinity of parts; at 144b–c, *f* is part of F-ness if *x* has *f*, and this leads to a discrete infinity of parts; thus the being of Unity, to choose an example, may be regarded both as a part of Unity, and as a part of Being. In the second place, the use of "part" for diversity of character or aspect is neither ordinary language nor ordinary Platonic Greek. It derives directly from the Dilemma of Participation stated at 131a–c. Thus it is that at 142b–c it is inferred that if Unity partakes of Being (F-ness), it partakes of its own being (*f*) (142c1), and that the signification of "is" in the statement "Unity is" is not Being, but the being of Unity. The argument here is not that of the Eleatic Stranger in the *Sophist* (255c–d), since it is meant, not merely to distinguish Unity from Being, but from *its own* being. Unless the latter distinction is taken as self-evident, it is unintelligible apart from the Dilemma of Participation.[10]

If to say that Unity is is to say that Unity partakes of Being, and if this implies that the being of Unity is a part of Unity, the claim that Unity is a whole, of which the being of Unity is part, becomes intelligible. Part and whole had earlier been defined in terms of each other: parts are parts of a whole, and a whole is that

[8] 130b, cf. *Phaedo* 102d, 103b.

[9] *PP*, p. 116.

[10] The ground for the inference that if Unity partakes of Being, it partakes of its own being, is Parmenides' earlier argument at 131a–c that if things partake of characters they cannot partake of the whole character, with the result that part of the character must be in them. This implication is restated at 144c8–d4. If the line of argument is pursued, the being of Unity is not only a part of Unity but also a part of Being—the use of "part" found at 144c. See also *Sophist* 257c–258c.

from which no part is absent.[11] This account implies both containment and dependence: a part, to be a part, must be part of some whole; a whole, to be the whole that it is, must contain the parts it contains. Parmenides assumes that this definition is satisfied by the relation between Unity and its own being: the being of Unity cannot exist apart from Unity, nor can Unity exist apart from its own being. This reciprocal dependence involves containment: as parts are parts of a whole, so the whole contains its parts (144e9).[12]

Given the Dilemma of Participation and appropriate definitions, Parmenides' inference that Unity, if it is, is a whole and has parts, is valid. It implies that Unity, since its own unity is part of it, partakes of Unity, that is, that Unity is itself a member of the class of things it characterizes. This premise, however, is not stated but implied. By a Third Man Argument, Parmenides might have gone on to prove an infinite multiplicity of Unities; instead, he proves that Unity has infinitely many parts. The peculiar and aporematic quality of this reasoning is shown by the fact that it is debarred in Hypothesis III, which corresponds to Hypothesis II in that it considers the consequences for the Others of a Unity which is, and maintains that each of those others, as one, will be other than Unity, since otherwise they would not *partake* of Unity but rather *be* Unity, whereas nothing but Unity itself can be Unity. This implies that since Unity itself is Unity, it cannot partake of Unity; it is not one among many things which are one.[13]

> *Paraphrase* (142d–143a, on Infinity of Parts): If Unity is, each of the parts of Unity must have unity and being, and the least part, therefore, contains two parts; for each instance of unity is, and each instance of being is one. Therefore, Unity is unlimited in multitude.

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[11] 137c, cf. *Theaetetus* 205a, and *Meta.* V 1023b26, *Phys.* III 207a10.

[12] If this is a correct account of Parmenides' use of the part/whole distinction here, it prompted debate in the Academy, echoes of which may be overheard in Aristotle's remark at *Categories* 1a24–25 that by "in a subject" he means what is *not* part of a subject and cannot exist separately from what it is in. Note that an Aristotelian substance cannot be a whole of which items in categories other than substance are parts, since if part and whole are interdependent, change in the parts implies change in the whole—a doctrine compatible with perpetual flux, but not with the notion of substance as self-subsistent substrate of change.

[13] 158a. The significance of this passage for the Third Man and the theory of Ideas generally was pointed out by H. F. Cherniss in "The Relation of the *Timaeus* to Plato's Later Dialogues," reprinted in *Studies in Plato's Metaphysics* (ed. Allen) (London, 1965), pp. 370–71 and n. 3 *ibid.*

This proof that Unity, if it is, has infinitely many parts, follows from what has gone before in a simple and cogent way. Whatever is is one, and whatever is one is. Therefore, the two parts of Unity, namely its being and its unity, must, respectively, be and be one. Thus, by the same argument which showed that Unity, if it is, has unity and being as parts, each of those parts, since they are and are one, must have unity and being as parts, *ad infinitum*. Assuming the transitivity of the *part of* relation, so that, for example, the unity of the being of Unity is part of Unity because it is part of the being of Unity, it follows that Unity has infinitely many parts.

The conclusion that Unity is "unlimited in multitude" is ambiguous between a process and a product. Does Parmenides mean that Unity *has* infinitely many parts, that it is infinitely divided? Or does he rather mean that it is (not infinitely divided but) infinitely divisible, that the *process* of division can be carried out on any part, however small? Ignoring the specifically Aristotelian connotations of the terms, this distinction is the distinction between an actual and a potential infinite.[14]

If this question were to be answered by analogy with Greek mathematics, as known to us through Euclid, writing about 300 B.C., there would be a strong case for treating Unity as a potential infinite. *Elements* X. 1. states that:

> Two unequal magnitudes being set out, if from the greater there be subtracted a magnitude greater than its half, and from that which is left a magnitude greater than its half, and if this process be repeated continually, there will be left some magnitude which will be less than the lesser magnitude.

This theorem, which later came to be called the Postulate of Archimedes, is the foundation of much Greek mathematics. Euclid's analysis of incommensurability and irrationality in *Elements* X, and the great Method of Exhaustion expounded in *Elements* XII, which anticipated the calculus,[15] rest on the theorem. It was known to, and probably first formulated by, Plato's contemporary and colleague

[14] Aristotle himself distinguishes his notion of a potential infinite from his usual notion of potentiality, precisely on the ground that the potential infinite cannot in principle be actualized. *Phys.* III 206a19–23.

[15] For a clear and concise statement of the connection, and the differences, between the calculus and the Method of Exhaustion, see C. B. Boyer, *The Concepts of Calculus* (New York, 1949), pp. 33–37.

in the Academy, Eudoxus. It will be observed that the theorem's treatment of infinity is purely potentialist: the theorem does not state that any magnitude is infinitely divided; it only states that any magnitude is infinitely divisible, as having no parts which do not admit the process of division. As with magnitude, so with multitude. Euclid's proof that (as we should be pleased to put it) there are infinitely many primes states rather that, "Prime numbers are more than any assigned multitude of prime numbers."[16] Aristotle correctly reported the best mathematical thinking of his time when he said that the mathematicians, "do not need the (actual) infinite and do not use it."[17] It is worth adding that an actual infinite is not implied by Plato's account of (phenomenal) infinity at *Philebus* 24a–25a, and may well there be precluded; that dialogue was written, as the *Parmenides* probably was not, after Eudoxus had joined the Academy.

But an actual infinite does seem implied by Parmenides' account here. Importantly, although Unity has in effect been proved to be infinite by division, there is no reference to the *process* of division; there is only mention of parts. Then again, it is said that the least part[18] comes to be from or is composed of two parts (142e4–5),[19] and that whatever comes to be a part contains or holds (ἴσχει, 142e4, 6, 7) two parts, namely unity and being. This suggests, not that the presence of parts arises from the process of division, but that the process of division is possible because of the presence of parts. It suggests, in short, an actual rather than a potential infinite, and this suggestion is confirmed by the fact that there are infinitely many numbers, each of which has being (144a–c).

Cornford suggested that the infinity proved in Parmenides' argument is somehow mental rather than real: it is shown that we can *think* of Unity as infinite, not that it *is* infinite.[20] But this is founded on nothing in the text, and is surely mistaken. Parmenides

[16] *Elements* IX. 20.

[17] *Phys.* III 207b29.

[18] The Greek superlative is often equilvaent to a very strong comparative, as indeed is true in English. There is no contradiction in saying that the least part has parts, understanding by this that the least part you choose has parts.

[19] For parallel construction with similar implication, see *Philebus* 27a11, *Sophist* 263d3.

[20] *PP*, p. 139.

claims, not that it may be thought of Unity that it has infinitely many parts, but that it may *truly* be thought of Unity that it has infinitely many parts; he had earlier argued that a thought must be a thought *of* something, and of something which is (132b–c). This, indeed, may well have been one motive for assuming an actual infinite, for Aristotle knew as an argument for an actual infinite that "it never gives out in thought."[21] That is, the act of distinguishing parts is merely recognition of their distinction.

Parmenides' proof that Unity has infinitely many parts satisfies one of Aristotle's definitions of continuity: "By continuous I mean that which is divisible into divisibles that are infinitely divisible."[22] This account implies, as Aristotle points out, that no two divisions in a continuum can be in succession; that is, given a point of division, there is no next or consecutive point of division, since between any two points of division there is a further point of division. As Aristotle puts it,[23]

> Nor, again, can a point be in succession to a point or a moment to a moment in such a way that length can be composed of points or time of moments: for things are in succession if there is nothing of their own kind intermediate between them, whereas that which is intermediate between points is always a line and that which is intermediate between moments is always a period of time.

This account of continuity in terms of infinite divisibility emphasizes its plurality. Aristotle gives another account which emphasizes its unity:[24]

> The "continuous" is a subdivision of the contiguous: things are called continuous when the touching extremities of each become one and the same and are, as the word implies, contained in each other; continuity is impossible if these extremeties are two. This definition makes it plain that continuity belongs to things that naturally in virtue of their mutual contact form a unity.

Although Aristotle argues that sameness of extremity implies lack of successiveness,[25] he nowhere argues that lack of successiveness implies sameness of extremity. But since both of these formulae are stated

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[21] *Phys.* III 203b22–25, cf. 208a15–19.

[22] *Phys.* VI 232b24–25, cf. 231b15–16, I 185b10, *de Caelo* I 268a6–7. See also the definition of a part as that into which anything can be divided, *Meta.* V 1023b13.

[23] *Phys.* VI 231b6–9.

[24] *Phys.* V 227a10–15.

[25] *Phys.* VI 230a24–29, *et seq.*

as definitions, and definitions of the same thing, Aristotle presumably supposed them to be equivalent.[26]

It is easy to construct an argument to show that they are not equivalent, since infinite divisibility is implied by but does not imply continuity. If, to use language no Greek could have used, the points on a given line are allowed to stand in one-to-one correspondence to the rational and real numbers, then the rational points on the line, corresponding to the Cantorian sequence 1, 2, $\frac{1}{2}$, $\frac{1}{3}$, 3 . . . , will be dense on that line; that is, between any two rational points there will be rational points. But density is not equivalent to continuity, since there are also irrational points on the line, points corresponding, for example, to the square roots of the primes; and as it happens, the irrational points are also dense on the line. Thus, infinite divisibility according either to a rational principle of division or an irrational principle of division does not imply continuity: continuity requires that both kinds of division be possible.

As a criticism of Aristotle's equivalence between continuity and infinite divisibility, this argument is beside the point. Aristotle lacked the notion of a number continuum, and could not have put the point numerically. But the geometry of his time was fully adequate to establish its analogue: Aristotle knew that a line could be divided infinitely according either to a rational or an irrational proportion, that is, divided in a proportion which is that of an integer to an integer or a proportion which is not. But to put the point thus geometrically is to alter it. Aristotle, and Greek mathematicians generally, would have understood that rational fractions, and *a fortiori* irrationals, are not numbers but ratios; they correspond, not to points on a line, but to ratios of lengths of lines. Thus Aristotle does not say that points are dense on a line, but that points are never in succession because there are always lines between them. Any line, of course, may be divided according to any ratio. Infinite di-

[26] One might argue that, so far from being equivalent, they are incompatible. For infinite divisibility implies continuity, and continuity implies the denial of successiveness, whereas sameness of extremity is taken to be a species of contiguity, and contact is defined as a species of succession. (*Phys.* V 227a9, cf. 17–24, *Meta.* XI 1068b26–1069a15.) The explanation is that Aristotle uses "in succession" in a strong and a weak sense. In the weak sense, two things are in succession if they may be taken in any order of prior and posterior; in the strong sense, that order must be a *discrete* order of prior and posterior. See *Phys.* V 226b34–227a6.

visibility, then, implies continuity because it implies division according to a ratio of things which are continuous. It follows that infinite divisibility is not the essence of continuity, but a convertible property of it. Aristotle's alternate formula, "things are continuous whose extremities are one,"[27] though it has other difficulties, is perhaps a better candidate for definition.

Aristotle, in discussing continua, has in view extensive magnitudes such as space, time, and motion, and Parmenides will shortly go on in Hypothesis II to deduce that Unity occupies space and time, and is in motion. Shall he then be supposed to have here proved that it is continuous?

Unity has not been shown to be continuous in the sense that the touching extremities of its parts are one and the same, for its parts have not been shown to have extremities, let alone identical extremities. But Unity has been shown to be divisible into parts which are infinitely divisible. This, by Aristotle's account, is both a necessary and sufficient condition for continuity. It should follow that Unity, satisfying this condition, is continuous.

However, it does not follow. Aristotle, dealing with extensive magnitudes, could assume that if they were divisible in a rational proportion, they would be divisible in an irrational proportion. But Parmenides, who has not shown that Unity is an extensive magnitude, provides an analysis which is restricted to two terms, being and unity, and limited to division answering to the simplest of rational proportions, the ratio of 1 to 2. The parts of Unity, therefore, stand to each other only in the ratio of a number to a number; the infinity derived is the infinity of multitude, not magnitude. A. E. Taylor, indeed, claimed that, "The 'rational fractions' are, to be sure, not a continuum, but they satisfy the only condition for a continuum known in Plato's time, that between any two a third can always be inserted."[28] But this claim founders on the fact of incommensurability, a fact of which Parmenides is well aware, since he has already given an elegant account of it.[29] Unity cannot be shown to be continuous by proving infinite divisibility according to a rational principle of division.

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[27] *Phys.* VI 231a21.
[28] *Plato* (New York, 1950), p. 511, n. 1.
[29] 140b–c, cf. *Theaetetus* 147d–148b, *Laws* VII 819e–820c.

Neither can it be shown to be continuous by assuming that infinite divisibility implies infinitesimal parts. For even if there were an implication between infinitesimal parts and continuity, Parmenides' argument, so far from implying a theory of infinitesimals, implicitly contradicts it: *if* a theory of infinitesimals was current in the Academy, it was based on a doctrine of indivisible lines,[30] or, generalized, a doctrine of minimal and indivisible parts, and the probable author of the doctrine was Xenocrates. Parmenides does not suppose that Unity has minimal parts, for he explicitly holds that the least part, i.e., the smallest part you take, has two parts, consisting of unity and being: every part of Unity is divisible, since every part is and is one. There can be no last term in an infinite sequence.[31]

Unity, then, has not been proved continuous. It has, however, been proved to have the most general feature belonging to any continuum, since it is a dense infinite, and not successive or discrete.

## II

*Paraphrase* (143a, the Plurality of Unity): Unity, since it is, has a share of Being, and thus is many. But let us now consider Unity by itself, apart from that of which it has a share. Is it only one, or is it also many?

Unity as a discrete infinite now becomes the subject for consideration. The question immediately raised is pursued without interruption to the end of the discussion at 145a. As a thing which is, Unity has already been shown to be many as containing infinitely many parts: it has the dense plurality characteristic of a continuum, though it is not itself continuous. But that argument rested on the assumption that, because Unity partakes of its own being, it is not the same as its own being (142b8–c1, cf. 158a), which is part of it. We are now asked to consider Unity in abstraction from its own being, and to determine whether, so conceived, Unity is one or many. The argument which follows will prove that it is many: it has the plurality proper to what possesses infinitely many discrete parts.

[30] *Meta.* I 992a20–22. See H. F. Cherniss, *Aristotle's Criticism of Plato and the Academy*, Vol. I. (Baltimore, 1944), pp. 14, 127–29, nn. 81, 322 (hereafter abbreviated *ACPA*).

[31] See also 132a–b, 133a, 165a–b; cf. 164c–165a, and M. Schofield, "The Dissection of Unity in Plato's Parmenides," *Class. Philol.* LXVIII (1972), n. 13.

The method of proof is to show that to distinguish Unity from its own being implies the existence of number; to infer that the numbers, of which there are infinitely many, exist or have being; that they therefore must have unity; and to conclude that Unity, considered apart from its own being, is distributed to all the numbers, and hence has infinitely many parts. It will be seen that the abstraction of Unity from its own being anticipates the reasoning of Hypothesis V, where it is argued that in speaking of a Unity which is not, we speak of something knowable and of something different from other things (160c–d), and that such a Unity must therefore in some sense be (161e–162b).

> *Paraphrase* (143b, Unity, Difference, and Being): Unity is not its own being, but different. If it shares in Being, it is not different from its being by virtue of being one, nor is its being different from Unity by virtue of Being; they are different by virtue of Difference. So Difference is not the same as Unity or Being.

This argument rests on the claim, first stated at 139c, that Unity, as such, cannot be different from anything, since it does not pertain to a thing *qua* one to be different from anything, but only to a thing *qua* different. Here, since Unity is not Being, but shares in it, its being is different from it; but it does not pertain to a thing *qua* being, any more than it pertains to a thing *qua* one, to be different; it pertains only to a thing *qua* different, and what is different is different by virtue of Difference (143b5).[32] This account may be compared to the arguments by which Being, Sameness and Difference are distinguished in the *Sophist* (255d–b), where, in conclusion, the Eleatic Stranger says of Difference that, "this nature pervades all the Forms; for each

[32] On this foundation, Parmenides will later erect an argument that since Unity is different from the others, and the others different from Unity, Unity is in this respect the same as the others and the others the same as Unity (147c). The argument is superficially paradoxical, but sound none the less: things are said to be the same when they partake of the same character; Unity and the others partake of the character of Difference; they are thus in this respect the same; they therefore also partake of Sameness. Similarly, Parmenides had argued in Hypothesis I (139d–e) that Unity cannot be the same as itself, since if it were, it would not be (just) one; Unity, if it has no parts, cannot partake of Sameness. Richard Robinson (*Plato's Earlier Dialectic*, 2nd ed. (Oxford, 1953), pp. 248–49) dismisses both arguments as willful sophisms. Both in fact depend on the assumption that Sameness and Difference are characteristics distinct from Unity.

is different from the rest, not by virtue of its own nature, but because it partakes of the character of Difference" (255e).

> *Paraphrase* (143c–d, Numerability): If someone were to pick out Being and Difference, or Being and Unity, or Unity and Difference, in each case he would pick out something properly called *both*. For it is possible to mention Being, and to mention Unity, and thus to mention each of two; and to mention each of two is to mention both. But to mention both is to mention two. Also, each of two is one, and if any one whatever is added to any pair whatever, the sum is three. Three is odd, and two even.

The exact force of this argument cannot be reproduced in English. Greek possesses, as English does not, a dual as well as a singular and plural inflection. When Parmenides argues that since it is possible to mention Unity and to mention Being, "each of two" has been mentioned, the English "two" is more explicit than the text, which contains only the genitive dual, αὐτοῖν. It is from this feature in the syntax of his language that Parmenides goes on to infer that both have been mentioned, and that since both have been mentioned, two have been mentioned.

It is important to remember, here and in what follows, that "number," ἀριθμός, means the natural numbers or positive integers; it is not by accident that the verb ἀριθμεῖν means "to count." Number was typically defined as "a multitude of units,"[33] a definition which implies that one is not a number, since it is not a multitude.[34] Aristotle, reflecting the mathematical tradition of his time, held that one is the beginning and measure of number, rather than a number, and explained this as follows:[35]

> . . . "the one" means the measure of some plurality, and "number" means a measured plurality and a plurality of measures. (Thus it is natural that one is not a number; for the measure is not measured, but both the measure and the one are starting-points.)

This does not mean, of course, that one was not used in mathematical operations: 6 + 1 = 7 is taken, not as the addition of a

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[33] *Elements* VII Def. 2. See also T. L. Heath, *The Thirteen Books of Euclid's Elements*, vol. ii. (New York, 1956), p. 280. (Hereafter abbreviated *TBE*.)

[34] The first person to treat one as a number, as distinct from *using* it as a number, appears to have been Chrysippus who appears, however, to have been anticipated by Speusippus: see *ACPA* n. 202; T. L. Heath, *History of Greek Mathematics*, vol. i. (Oxford, 1921), p. 69 (hereafter *HGM*); W. D. Ross, *Aristotle's Physics* (Oxford, 1936), p. 604.

[35] *Meta.* XIV 1088a5–7, cf. V 1016b18ff., 1021a13.

number to a number, but of a unit to a number. Plato consistently defines arithmetic as the science of odd and even,[36] and though he nowhere explicitly states that one is not a number, this is implied by this passage.[37] Two is the first number:[38] and it is by establishing that there are items numberable by the first number, rather than by one, that Parmenides begins his account. It will be observed that the assumption that Unity, taken apart from its own being, is numerable, implies that it in some sense is, an anticipation of the argument of 161e–162b.

> *Paraphrase* (143e–144a, the Numerical Infinite): If there are two things, there is *twice*, and if three things, *thrice*, since two is twice one and three is thrice one. Therefore, there is twice three and thrice two. Thus there are even-times even numbers, odd-times even numbers, even-times odd numbers, and odd-times even numbers. If this is so, no number is left remaining which does not necessarily exist. Thus, if Unity is, number necessarily also is. And if number is, there are many things and an unlimited multitude of things which are, since number is unlimited in multitude and has a share of Being.

Parmenides had begun by distinguishing Unity and Being; but before attempting to show that a Unity which is, when considered apart from its own being, implies the existence of infinitely many numbers, he had distinguished Difference in addition. The reason for this undoubtedly comes from number theory; not only two, the first even number, but three, the first odd, is required for the classification of numbers which he here introduces.[39]

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[36] *Theaetetus* 198a, *Gorgias* 453e, cf. *Phaedo* 104a–b.

[37] Cf. *Republic* VII 524d, *Phaedo* 104a–b. *Hippias Major* 302a treats one as an odd number, but this is dialectical, and does not imply a commitment to the proposition that one is odd as a matter of abstract number theory.

[38] Cf. *Meta.* XIII 1085b 10, *ACPA*, p. 202. Even here, the ancients were not unanimous, sometimes regarding two not as a number, but as the beginning of even number. See *HGM*, pp. 70–71. One argument for this was that it is characteristic of numbers that their product is greater than their sum, whereas $2 + 2 = 2 \times 2$, and $1 \times 1$ is less than $1 + 1$. See M. L. D'Ooge, *Nicomachus: Introduction to Arithmetic* (New York, 1926), p. 116ff.

[39] The initial numbers in Parmenides' classification are 4, the first even-times even number, 6, the first even-times odd or odd-times even number, and 9, the first odd-times odd number. 1, the measure of number, and 2 and 3, the first even and odd numbers, respectively, have already been introduced. These numbers, and the classification which they represent, were of considerable importance in Greek number theory. See *Elements* VII Defs. 8–10 for the classification; for further discussion, see *HGM*, i pp. 71–72, and *TBE*, ii pp. 281–84.

Parmenides' classification of the numbers has often been supposed to be equivalent to an account of their generation.[40] This claim appears to be as old as Aristotle;[41] it is, however, mistaken.

The term "number," as Aristotle points out, has two senses: that by which we count, and that which is counted or countable.[42] Parmenides, if he were a Platonist (cf. 135b–c), would suppose that that with which or by which we count are Idea Numbers such as Two, Three, Four, and so on,[43] a serial plurality which is itself countable. These Ideas are not pluralities of units, but the number-properties of such pluralities; they cannot be generated by arithmetical operations such as addition and multiplication, since such operations presuppose their existence. Indeed, as Ideas, they cannot be generated at all.

Parmenides, then, proceeding by addition and multiplication, has not generated numbers as properties which numbered pluralities possess; has he, then, generated numbers as pluralities which possess those properties? The answer is that he has not. If a given number is a plurality of units, then to derive that number is to show that there are as many units as that number. The multiplication used in "twice two" requires an existence assumption, namely, that four is a number, that is, that there are four units; four cannot be three units, one of which is counted twice. But Parmenides, deriving numbers successive to three by simple multiplication, does not attempt to show that there are pluralities of units corresponding to the products of multiplication. He might easily have done so if he wished; for he has available his previous proof that Unity has infinitely many parts, and what is infinitely divisible is infinitely numberable.

The truth is that Parmenides has presented, not a generation,

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[40] For further discussion, see "The Generation of Numbers in Plato's Parmenides," *Class. Philol.* LXV (1970), pp. 30–34. Part of the argument is here reproduced, in revised form.

[41] *Meta.* I 987b 32–34, cf. *ACPA* n. 106. At XIII 1084a 2–6, Aristotle gives an account of the generation of number which resembles Parmenides' argument, and is perhaps a misinterpretation of that argument.

[42] See *Phys.* IV 219b5–8.

[43] See *Phaedo* 96e–97b, 101b–c, 104a–b (comparing with the latter *Post. Anal.* II 96a24–96b14). See further J. Cook Wilson, "On the Platonist Doctrine of the ἀσύμβλητοι ἀριθμοί," *Class. Rev.* XVIII (1904), pp. 247–60; H. F. Cherniss, *The Riddle of the Early Academy* (Berkeley and Los Angeles, 1945), pp. 33–37 (hereafter *REA*), and *ACPA*, pp. 300–05 and Appendix VI.

but an existence proof, resting on the assumption that if $m$ and $n$ are integers, the product and sum of $m$ and $n$ are integers.[44] It will be seen that any of Parmenides' methods—multiplication of even numbers, of odd numbers, or of odd and even numbers—will suffice to prove the existence of any number, since any of these methods, by repeated application, will prove the existence of a number larger than any number desired. Parmenides introduces four methods, rather than one, in order to provide not only an existence proof of numbers, but a classification of them.[45] Nothing in this argument indicates that numbers can be constructed or derived from simpler constituents, or generated in any way. Parmenides' argument is compatible with the view that numbers are timeless objects which are neither generable nor destructible; it is also compatible with the view that numbers are simple essences incapable of analysis into ontologically (as distinct from numerically) prior and posterior elements. The purpose of Parmenides' argument is not to provide a *ratio essendi* for number, but a *ratio cognoscendi.* If Unity exists, then, since it can be considered apart from its own being, it implies the existence, not only of Being, but of Difference. There is, then, a plurality with these members; and to recognize the existence of that plurality is to commit oneself to the truths that $2 = 2 \times 1$ and $2 + 1 = 3$. But to accept any mathematical truth is to accept every mathematical truth;

[44] The objection that Parmenides' procedure does not generate the primes, which is as old as Aristotle (*Meta.* I 987b 33) neglects the fact that multiplication is abbreviated addition (*Elements* VII, Def. 15, cf. *TBE* ii, p. 287), and also the fact that no generation is here involved. Cornford's explanation of the point is mistaken: "The objection that prime numbers cannot be obtained by multiplication is invalid, since Plato evidently includes addition and starts with that when he *adds* one term to another to make two, and two to one to make three. Moreover, primes were sometimes regarded as odd multiples of an odd number, 1 being treated for this purpose as odd: $5 = 5 \times 1$" (*PP*, p. 141, n. 2). Cornford goes on to cite Theon (*Plat. Arith.* 23, 14) in support of the last remark. But he forgets that one was *not* added to one to make two, two rather being derived from the dual and shown to imply two ones; that the passage from Theon which he cites also says that a prime is "measured by no number, but by a unit alone" (23, 9–10); and that to analyze 5 as the product of 5 and 1 requires a number system which already contains 5.

[45] The classification is not exhaustive, since it does not classify the primes, and not exclusive, since even-times odd and odd-times even numbers are the same, and some numbers may be both even-times odd and even-times even: $12 = 2 \times 6 = 4 \times 3$. There was, however, an ancient tradition that even-times even numbers are always (in effect) numbers of the form $2^n$: see *TBE* ii, p. 282.

the existence of any number implies the existence of every number. Therefore, the existence of a plurality with three members implies the existence of a plurality with infinitely many members, namely, the plurality of numbers. The effect of Parmenides' argument is to establish precisely the implication that Proclus, long afterward, claimed for it: "If One exists, number will exist, from which it follows that (infinite) plurality exists."[46] This implication is explicitly stated in the *Sophist* (238a–b, cf. *Theaetetus* 185d). In short, the hypothesis that Unity exists implies the existence of the number series; but an existence proof is not a generation.

Aristotle in the *Physics* maintains that, "Everything that is infinite may be so in respect of addition or division or both."[47] By what is infinite by division, Aristotle understands, as we have seen, what is continuous, divisible into divisibles which are infinitely divisible. He nowhere explains what he means by the expression "infinite by addition," but it clearly applies to the infinity which is characteristic of numbers. This is confirmed by his claim that the infinite by division and the infinite by addition are in a sense the same:[48]

> In a way the infinite by addition is the same thing as the infinite by division. In a finite magnitude, the infinite by addition comes about in a way inverse to that of the other. For in proportion as we see division going on, in the same proportion we see addition being made to what is already marked off. For if we take a determinate part of a finite magnitude and add another part determined by the same ratio (not taking in the same amount of the original whole), and so on, we shall not traverse the given magnitude.

That is, what is infinitely divisible is infinitely numerable; to the series of divisions there corresponds the series of numbers. It will be recalled that "number" means the natural numbers, exclusive of one (except for counting) and zero;[49] and if number is so defined, the number series is not continuous, but rather, as Aristotle points out,[50] successive or discrete. A thing is in succession when:[51]

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[46] *Schol. in Parm.* 1261, 18–21 (Cousin).

[47] *Phys.* III 204a7, cf. 206a15.

[48] *Phys.* III 206b3–10.

[49] A. E. Taylor (*Plato*, pp. 505–06) attempted to give Plato the zero, and claimed to find the real numbers at *Epinomis* 990c–991b (p. 501, n. 1). The latter text, however, limits number to the odd and the even.

[50] *Phys.* V 227a20, *Meta.* XIII 1085a4.

[51] *Phys.* V 226b34–227a2, cf. *Parmenides* 149a.

> it is after the beginning in position or in form or in some other respect in which it is definitely so regarded, and when further there is nothing of the same kind as itself between it and that to which it is in succession.

Thus for example, there is no natural number between any two natural numbers which are in succession. Parmenides will shortly go on to show that Unity has parts corresponding to each of the numbers, and this is equivalent to showing that Unity has infinitely many discrete parts.

> *Paraphrase* (144a–b, the Actual Infinite): If number has a share of Being, each part of number has a share of Being; therefore, Being is distributed to all of the many things which are, and is lacking to none of them; for Being is lacking to nothing which is.

As before, in the case of the infinite by division, this argument would seem to imply an actual rather than a potential infinite. Parmenides supposes that since number, which is unlimited in multitude, has a share of Being, *each* number therefore has a share of Being. He will go on to infer that the parts of Being are therefore unlimited in multitude. To say that number, taken collectively, is unlimited in multitude is compatible with the claim that numbers are merely potentially infinite in that for any number chosen there is a greater number. But Parmenides' argument construes number distributively; it would seem therefore to imply that there are (actually) infinitely many numbers.

Granting that there is an infinite plurality of numbers, and that this plurality is to be taken as actual, the question arises whether there is a number of that plurality, i.e., an infinite number. Aristotle provides evidence that someone in the Academy had formulated this conception;[52] nevertheless, it is not to be found here. Parmenides does not refer to an infinite number, but to number infinite in multitude (144a6). It is not self-contradictory to suppose that there are infinitely many numbers, each of which is finite, unless one supposes that "infinite" is itself a number adjective, which is precisely the point at issue. Again, it is not self-contradictory to suppose that a thing has infinitely many parts, but no parts which are "infiniteth" in order of distinction.

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[52] *Meta.* XIII 1083b37–1084a9, cf. *Phys.* III 206b7–9. If Xenocrates formulated a doctrine of infinitesimals, the conception should be attributed to him, since infinitesimals logically presuppose an infinite (ordinal) number.

*Paraphrase* (144b–c, the Parts of Being): Being is distributed to all of the many things which are, from the smallest to the greatest; for Being is not lacking to anything which is. Being is divided up among things in every way, from the smallest to the greatest. It is of all things most divided, and there are parts of being without limit. Therefore, its parts are most multitudinous.

The conclusion of this argument, that the parts of Being, since they are unlimited in number, are most multitudinous (πλεῖστα, 144c1, 2), raises a question which cannot on the basis of this passage be answered. The conclusion is not directly derived from the premise that Being is not lacking to anything which is (144b3–4), since that premise is used in a sub-argument to show that Being is not lacking to any of the numbers, of which there are infinitely many (144b4–6), and it is from the latter that the conclusion that the parts of Being are most numerous is derived. The conclusion, therefore, appears to be derived *simply* from the infinity of number, despite the fact that the existence of other things besides number, Being and Difference, for example, has already in Hypothesis II been proved. Again, the parts of Being are distinguishable from the numbers whose being they are by the same reasoning which allowed Unity to be distinguished from its own being. This raises the question whether Parmenides, in inferring that the parts of Being are *most* multitudinous, is assuming that if $X$ is an infinite plurality, no plurality is greater than $X$.[53]

This assumption implies a paradox: the paradox, to express it in set-theoretical language, that an infinite set may be equivalent to a proper subset of itself, as the set of integers is equivalent to the set of even integers, or the set of points on a line equivalent to the set of points on the half-line. That there should be pluralities which may be matched member for member, and yet which are in the customary sense of containment unequal, is an implication of an actual infinite, and a paradox without analogy to the behavior of finite and familiar sets. It was not until the nineteenth century that this paradox was relieved of the appearance of absurdity.[54] Thus

[53] Notice, however, that 149b–c contradicts this assumption, if it is supposed that there are infinitely many contacts.

[54] See B. Bolzano, *The Paradoxes of the Infinite* (trans. D. A. Steele) (London, 1948), pp. 98 *et seq.*

it is that Plutarch can state as an objection against the Stoic doctrine[55] of infinite divisibility that, if it were true, it would imply that,[56]

> the man is not composed of more parts than the finger, nor the universe of more parts than the man, for physical division goes on to infinity; but among things infinite there is neither a greater nor a less. . . .

This implication is rejected as obviously absurd, and, though the texts are silent, this may explain why, if the *Parmenides* assumes an actual infinite, it was the Aristotelian and Eudoxian potential infinite which made its way in mathematics. The doctrine of the potential infinite is not a dogma in metaphysics, but a solution in mathematics: if it is absurd to say that the half-line has as many parts as the line, it is not absurd to say that any operation of division carried out on the line may proportionally be carried out on the half-line.[57] The stone which was to become the foundation of the modern treatment of infinity was originally rejected by the builder. It may first have been rejected in the Academy.

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[55] Perhaps that of Chrysippus. See Diog. Laert. VII, 150, and Stob. *Ecl.* L. 142, 2 cited by H. von Arnim, *Stoicorum Veterum Fragmenta*, vol. ii. (Berlin, 1903), p. 158, para. 482.

[56] Plutarch, *De Comm. Not.* 1079a, cited by von Arnim, *op. cit.*, p. 159, para. 484. It has been held that in this passage it is, "clearly stated that the subset is equivalent to its set in the sense defined by the modern theory of sets." (S. Sambursky, *The Physics of the Stoics* (London, 1959), p. 97; cf. A. Wedberg, *Plato's Philosophy of Mathematics* (Stockholm, 1955), pp. 65, 130; W. K. C. Guthrie, *History of Greek Philosophy*, vol. II. (Cambridge, 1965), p. 290, has traced the assumption back to Anaxagoras). But the Stoics themselves refused to say that any body or continuum consists of an infinite number of parts, so that we here deal not with an accepted theory of infinity, but with an acknowledged mistake. I am indebted to H. F. Cherniss for this point, which will be explained in greater detail in his forthcoming edition and translation of Plutarch's "On Common Notions."

[57] Sir Thomas Heath remarks: "The Greek geometers shrank from the use of such expressions as infinitely great or infinitely small and substituted the idea of things greater or less than any assigned magnitude. Thus . . . they never said that a circle is a polygon with an infinite number of infinitely small sides; they always stood still before the abyss of the infinite and never ventured to overstep the bound of clear conceptions. They never spoke of an infinitely close approximation or a limiting value of the sum of a series extending to an infinite number of terms," *Archimedes* (Cambridge, 1897), p. cxlii. But if an actual infinite is contemplated in the *Parmenides*, and if a doctrine of infinitesimals and infinite number was held in the Academy, the fact that subsequent Greek mathematicians "stood still before the abyss of the infinite," even though they used heuristic methods which suggested that they might advance, was the product of decisive rejection.

We have already found reason to suppose that Parmenides has assumed an actual infinite. And if he is here assuming that there is no plurality greater than an infinite plurality, this would explain why he takes such pains to stress that the parts of being are *most* multitudinous. But it is to be remembered that the Greek superlative is often equivalent to a strong comparative,[58] and so the question cannot be settled on the basis of this text. It is to be observed, however, that the force of πλεῖστα, with its repetition, is emphatic (144c1, 2, cf. 144d6).

Questions of infinity aside, perhaps the most arresting feature of Parmenides' argument is the shift it involves in the use of the word "part." Previously, if Unity partook of Being, its own being was part of it. We now find a use in which a given number is part of number, and the being of that number part of Being. These are distinguishable uses. For plainly, "all number" at 144a8, which is equivalent merely to "number" at 144a4, is not the characteristic of being a number; it is the number series itself. But the basis of the inference that Being is distributed to the numbers, and therefore has parts, is the proposition that, "if all number partakes of Being, each part of number will partake of it too" (144a6–8), where each part of number means the individual numbers. This conclusion can only follow from the Dilemma of Participation, which implies that the being of any number is not only part of that number, but part of Being.

> *Paraphrase* (144c–d, the Division of Unity): Nothing is part of Being but not *a* part. If a thing is, then so long as it is, it is one thing (ἕν); it cannot be no-thing (μηδέν). Unity is therefore present in addition to every part of Being, and is lacking neither to the smaller nor the larger part nor any other part. But Unity, as a whole, cannot be in many places at once; therefore it is divided. Only as divided can it be present to all the parts of Being at once.

The reason given for supposing that, if many things are one, Unity has parts, is also a reason for supposing that if many things

[58] Compare the use of τὸ ἐλάχιστον at 142e4 and of σμικρότατα καὶ μέγιστα at 144b5, cf. 2–3. See also the comparatives in 144b7–8. On the other hand, Aristotle (*Phys.* IV 220a26–33) refers to the smallest number: it is two. At *Phys.* III 206b27–33, Aristotle ascribes to Plato the belief that one is the minimum number, and ten the maximum. There is, however, no evidence in Plato for either claim, the latter of which is contradicted by 144a.

are, Being has parts. That reason is, in effect, one horn of the Dilemma of Participation.

The Dilemma of Participation (131a–c) is Parmenides' first direct criticism of the theory of Forms. In structure, the argument is a *reductio*: if there is participation, there is participation either in the whole Form or part of the Form; there can be participation neither in the whole Form nor in part of the Form; therefore, there is no participation. This *reductio* grounds Parmenides' later objection that if Forms exist, they have no relation to things in our world (133b–e) and are hence unknowable (134a–e). But though the Dilemma is in structure a *reductio*, an implication derives from difference in the modality of its disjuncts: if there is participation neither of whole nor part, there is no participation; but if there is participation, it cannot be of whole but must be of part.

The objection to the first disjunct is that if things partake of the whole Form, then the whole Form will be *in* each thing; but if so, then one and the same thing will be in many things at once, and so be separate from itself. Since the minimal meaning of "separate" here implies non-identity (cf. 129d7, 130b2, 3, 4, d1), this conclusion, if valid, is equivalent to a logical impossibility. The argument may be compared to Aristotle's proof that a universal, being common to many things, cannot be the substance of that to which it is common: for if it were the substance of *a* and the substance of *b*, then *a* and *b* would be identical, whence it would follow that the universal is not common after all and thus not universal.[59] Parmenides' argument is, as it were, the obverse of this: if *a* and *b* are not identical, and if both partake of the whole of a Form, then the Form cannot be identical with itself. The alternative is to suppose that the one Form is divided, and that things which partake, partake of parts of Forms—each partaking of *its own* part, lest the argument of the first disjunct apply.

Socrates rejects both disjuncts of the Dilemma, but the modality of those disjuncts is very different. The first, given plurality of participants, implies logical absurdity: the Form is not identical with itself, and this applies universally, to every Form. The second, whether true or (as Socrates thinks) false, is contingent. However odd it may seem to construe universality in terms of the relation of

[59] *Meta.* VII 1038b7–14, cf. 16–33.

part and whole, it is not thereby logically absurd. Thus the Dilemma, though a *reductio*, issues in an implication by reason of the difference in the modality of its disjuncts: if there is participation, it is participation in parts of Forms, not in wholes. This difference in modality is confirmed by *Philebus* 15b6–7.

The implication of the Dilemma plays an important role in the concluding part of the *Parmenides*. Hypothesis I proves that if Unity is not a whole of parts, no characteristic belongs to it.[60] Hypothesis II proves that if even one characteristic belongs to it—the bare characteristic of being—Unity is a whole of parts, and goes on to derive for it precisely the characteristics denied of it in Hypothesis I. The propositions that if Unity is not a whole of parts no characteristic belongs to it, and that if a characteristic belongs to it it is a whole of parts, are transpositionally equivalent, and presuppose that the assertion of a characteristic of a subject is to be construed in terms of the relation of part and whole. There can be but one justification for it within the body of the *Parmenides*, and that is the Dilemma of Participation, which implies that if there is participation it must be of parts. This implication explains why, if Unity partakes of Being, it partakes of its own being, which is part of it, and why the signification of "is" in the statement "Unity is" is not Being, but the being of Unity. It explains why Being is distributed to the numbers and divided, and why Unity as a whole cannot be in many places at once, but is present to all the parts of Being only as divided. Finally, it explains why the term "part" has a double force, so that the being of Unity can be treated as a part of Unity, and the being of a number as a part of Being. In short, the implication is repeatedly assumed as a premise in the detailed argumentation of 142c–145a.

The remark that Unity is "lacking neither to the smaller nor the larger part (of Being) nor any other part" is at first sight puzzling. The use of the comparatives is collective rather than distributive, and the force of the phrase is to add emphasis to the universality of the conclusion: Unity is not lacking to the parts of Being, however those parts may be ordered. A collection of parts less than the whole is still one collection.

[60] Cf. Hypothesis IV, 159c–d.

*Paraphrase* (144d–e, the Infinity of Unity): What is divided into parts is as many as its parts. Thus it is not true that Being has been distributed to the *greatest* number of parts: it has not been distributed to more parts than Unity, since nothing which is lacks unity, and nothing which is one lacks being. Therefore, Unity is distributed by Being, and is unlimited in multitude. Thus, not only is Unity as a thing which is (τὸ ὂν ἕν) many, but Unity itself (αὐτὸ τὸ ἕν), that is, apart from its own being, is many, since it is distributed by Being.

The type of infinity here in view is allied, obviously, to the successive infinite of number—allied, but not identical. Aristotle gives two criteria of successiveness: items in succession are ordered according to position, form, or some other principle; and there is nothing of the same kind between them. The numbers, standing in an order of prior and posterior, satisfy both requirements. But the being and the unity of the numbers, as parts of Being or parts of Unity, do not stand in an order directly determined by position or form: they are not in succession directly, even though that to which they belong is in succession. As the being or unity of things which are in succession, however, there is nothing of the same kind between them. Parmenides' earlier account of the infinite by division seemed to imply not continuity but density; the account here of the infinite by addition seems to imply, not successiveness but discreteness.

The conclusion of this argument, that not only is the Unity which is (or the being which is one) unlimited in multitude, but also that Unity itself, i.e., considered apart from its own being (cf. 143a), is unlimited in multitude, may well seem strange and paradoxical. It has been shown that to hypothesize a Unity which is, and then to consider it apart from and thus as different from its own being, implies the existence of the numbers, and that the numbers, because they partake of Being, partake of Unity; yet this can hardly be said to imply that Number, in partaking of Unity, partakes of Unity apart from its own being. But the aim of the argument is not to show that if the numbers are one, they partake of Unity conceived apart from its own being. It is to show that Unity, conceived apart from its own being, has parts if the numbers are one. In short, if Unity is participable, it is partable, whether or not it is said to be. This again is a direct consequence of the Dilemma of Participation.

*Paraphrase* (144e–145a, Conclusion): Because parts are parts of a whole, Unity will be limited with respect to its wholeness, since parts are contained by the whole, and what contains is a limit. Thus unity is whole and parts, limited and unlimited in multitude.

This conclusion is stated generally, as a consequence which obtains not only for Unity as a thing which is, but for Unity itself, that is, conceived apart from its own being. Unity as a participant was shown to be a whole of parts as 142c–d. It is now been shown to be a whole of parts as participable. As we have seen, this involves a shift in the use of the word "part," in that the part of Being or Unity which anything has is now regarded, not as a part of the thing, but as a part of Being or Unity. The coextensiveness of Being and Unity implies that anything which is is one, and that anything which is one partakes of Unity, and thus has a part of Unity. Unity is therefore a whole from which no part is absent, since there is nothing which is that does not have a part of it. Thus both as participable and participant, Unity is many, is a whole, and has parts; it is both limited and unlimited—limited, as being a whole, and unlimited as containing infinitely many parts.

So far we have followed a complex and tangled path to a decidedly queer result. The Form of Unity has been shown to be in two ways a whole of parts, respectively dense and discrete. This accords with the remark in the *Sophist* that, "If a thing is divided into parts, there is nothing against its having the property of unity as applied to the aggregate of all the parts, and being in that way one, as being a sum or whole."[61] The whole is taken to be a limit of its parts, as containing them, a result which anticipates Aristotle's dictum that the infinite is not what contains, but is contained.[62] In so far as we deal here with a thing merely taken as one and whole, there is no more difficulty in this than in Socrates' earlier claim (129c–d) that he is both one and many, an apparant paradox dismissed in the *Sophist* (251a–c) and described in the *Philebus* (14d) as childish.

But it is otherwise if the thing taken to be one and whole is the Form of Unity itself; for the *Sophist*, in the same passage in which it is said that a thing divided into parts may be one and whole, also says, "Surely Unity in the true sense and rightly defined must be altogether without parts,"[63] whence it follows that a whole of parts can have, but cannot be, Unity. Yet that we deal in 142b–145a

[61] 245a, trans. Cornford. Cf. *Theaetetus* 205a, *Parmenides* 157c–e.
[62] *Phys.* III 208a4. See also *ACPA*, pp. 110–11, and nn. 66, 67.
[63] *Sophist* 245a.

with Unity is shown by the fact that we deal with the characteristic which anything said to be one has. There is no ambiguity between the subjects of the first and second arguments, since the second argument consists merely in considering the subject of the first argument apart from its own being.

The only alternative to regarding Unity as a Form which can be justified from the text of the *Parmenides* is that the subject of the argument is the historical Parmenides' One Being. This is suggested by two passages: at 128a–b, Parmenides' hypothesis (cf. 128d) is said to be that the All is one; at 137b, Parmenides says that in playing the laborious game of dialectical exercise which is to follow, he will begin with *his own* hypothesis. This is evidence that the subject under discussion is the Parmenidean One. But not good evidence. Between 128 and 137, Socrates announces his own theory of Forms, and Parmenides offers a series of objections to it. It is too often overlooked that Parmenides holds that those objections are answerable by a man of natural gifts and sufficient intelligence,[64] and proclaims himself a convert to the theory, holding that to deny it is to destroy the significance of thought and discourse (135b–c). That the "one" of which he speaks is the character of being one, that is, the Form of Unity, is evident throughout the hypotheses which follow, and is implied by 135d–e and 136a–c. Parmenides, then, speaks as a convinced Platonist: his hypothesis is no longer that the All is one, since the theory of Forms is a pluralistic theory. His hypothesis is, or presupposes, that Unity is a Form, a character in which things partake. It is consistent with this that Parmenides' hypothesis that the All is one is criticized in the *Sophist* (244b–245e) by an Eleatic Stranger who uses argumentation closely similar to that found in the Hypotheses in the *Parmenides*; the similarity need not be surprising, for in respect to problems involving their being and unity, the Parmenidean One and the Platonic Form of Unity are in important respects homologous.

It follows, however, that the deduction in 142c–145a is of a highly unfortunate character, given that its subject is the Form of Unity. Socrates had earlier said that though he saw no difficulty in claiming that the things we see are both one and many, like and

[64] 135a–b, cf. 135c–d, 133b, 130e.

unlike, he would be astonished if it were shown that Unity is many or Plurality one (129b–130a). Parmenides has given ample reason for surprise. Hypothesis I proved that if Unity is not many, it can have no parts (137c–d), whence it is deduced that Unity cannot be even to the degree of being one, that nothing can be said of it, and that it is not an object of discourse (141e–142a)—a conclusion which ill comports with its (to misspeak) being a Form. Parmenides has now shown that if Unity partakes of Being it has infinitely many parts, and infinitely many parts even if taken separately from its own being. The inference that Unity may be a whole of parts, and thus many, contradicts Socrates' original theory, and is itself contradicted by *Sophist* 245a. The contradiction occurs again in the body of the *Parmenides* itself: for in Hypothesis III (158a–b) it is argued that since a whole is one whole and each part one part, they partake of Unity and therefore cannot be Unity, but are other than Unity. This implies that Unity cannot be a whole of parts. We are asked, in 142c–145a, to examine the logic of the argument and to inquire what is assumes; we are not asked to accept the truth of its conclusion, any more than we are expected to accept the truth of the ultimate conclusion of all the Hypotheses at 166c.

There is a further peculiarity. Parmenides has concluded that Unity is a whole of parts, and that those parts are contained by the whole, which is limited in respect to its wholeness. This is no trifling inference, since he will next argue that if Unity is a whole it must have extremities, and therefore beginning, middle, and end; so Unity has shape. The minimal condition on which Unity can have shape is that it is a continuous and extensive magnitude. There is nothing in 142c–145a to justify this claim directly; the argument so far has shown only that Unity must have multitude, not magnitude. If the inference is to be justified, it must be latent in premises other than those by which this argument explicitly proceeds. As has often been remarked, the Dilemma of Participation admits, though it does not require, a materialist interpretation:[65] if we assume that participants are in space, and that parts of Forms are in participants, it is an easy inference that parts of Forms are in space. So it is that Aristotle in the *Topics*[66] holds that if the Ideas are in us, they must be objects

[65] Cf. *Meta.* VII 1039a24–b19.
[66] *Top.* II 113a24–32, cf. *ACPA*, p. 532.

both of sensation and of thought, and are both in motion and at rest, "for when we move, it follows necessarily that all that is in us moves with us as well." These consequences are deduced for Unity in Hypothesis II; it remains to be shown how and in what way they are connected with the Dilemma of Participation, which requires fuller consideration than has been given to it.

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