On the Logic of the Ontological Argument*

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Saint Anselm of Canterbury offered several arguments for the existence of God. We examine the famous ontological argument in *Proslogium* II. Many recent authors have interpreted this argument as a modal one.[1] But we believe that Jonathan Barnes has argued persuasively that Anselm's argument is not modal.[2] Even if one were to construe the word 'can' in the definite description 'that than which none greater can be conceived' in terms of metaphysical possibility, the logic of the ontological argument itself doesn't include inferences based on this modality. In this paper, we develop a reading of Anselm's *Proslogium* that contains no modal inferences. Rather, the argument turns on the difference between saying that *there is* such a thing as x and saying that x has the *property* of existence. We formally represent the claim that *there is* such a thing as x

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[1]See the works by Malcolm, Hartshorne, Adams, Plantinga, and Lewis cited in the Bibliography.

[2]See Barnes, [1972], Chapter 1, §III. Morscher, in [1990], also takes the argument to be a non-modal one.

by '$\exists y(y{=}x)$' and the claim that x has the property of existence by '$E!x$'. That is, we represent the difference between the two claims by exploiting the distinction between quantifying over x and predicating existence of x. We shall sometimes refer to this as the distinction between the being of x and the existence of x. Thus, instead of reading Anselm as having discovered a way of inferring God's actuality from His mere possibility, we read him as having discovered a way of inferring God's existence from His mere being.

Another important feature of our reading concerns the fact that we take the phrase "that than which none greater can be conceived" seriously. Certain inferences in the ontological argument are intimately linked to the logical behavior of this phrase, which is best represented as a definite description.[3] If we are to do justice to Anselm's argument, we must not syntactically eliminate descriptions the way Russell does. One of the highlights of our interpretation is that a very simple inference involving descriptions stands at the heart of the argument.[4]

The Language and Logic Required for the Argument

We shall cast our new reading in a standard first-order language. It contains the usual two kinds of simple terms: constants $a_1, a_2, \ldots$ ($a, b, \ldots$ as metavariables) and variables $x_1, x_2, \ldots$ ($x, y, \ldots$ as metavariables). It also contains predicates $P^n_1, P^n_2, \ldots$ ($n \geq 1$) ($P^n, Q^n, \ldots$ as metavariables). In addition to atomic formulas of the form $P^n\tau_1 \ldots \tau_n$ and identity formulas of the form $\tau{=}\tau'$ (where the τs are any terms), the language consists of complex formulas of the form $\neg\varphi$, $\varphi \rightarrow \psi$, and $\forall x\varphi$.[5] We shall suppose that where φ is any formula, then $\imath x\varphi$ constitutes a complex, though primitive, term of the language. We read $\imath x\varphi$ as 'the (unique) x such

[3]D. Lewis, in [1970], says:

> We will also not have to worry about the logic of definite descriptions. If I say "That which is red is not green" I might just mean "Whatever is red is not green," neither implying nor presupposing that at least or at most one thing is red. Similarly, we can construe Anselm's "that, than which none greater can be conceived" not as a definite description but rather as an idiom of universal quantification.

This phrase might be construed as an idiom of universal quantification. Such a construal, however, would not capture its use by Anselm as a definite description.

[4]This inference is labeled *Description Theorem 2* in what follows.

[5]We frequently drop the quotation marks by which we mention expressions of our object language whenever our intent is clear.

that φ' and we refer to these terms as definite descriptions. Note that primitive descriptions (in complex formulas) do not have 'scope.' For example, in a complex formula such as $P\imath xQx \rightarrow S\imath xTx$, there is no option of asking whether the descriptions have wide or narrow scope (or primary or secondary occurrence), since the formula is not an abbreviation of, or eliminable in terms of, any other formulas.

In what follows, we use 'τ' to range over all terms: constants, variables, *and* descriptions. We use 'φ^{τ}_{x}' to designate the result of substituting term τ for each free occurrence of the variable x in formula φ.

The models of this simple language are standard. A *model* $\mathbf{M}$ is any pair $\langle \mathbf{D}, \mathbf{F} \rangle$, where $\mathbf{D}$ is a non-empty set and $\mathbf{F}$ is a function defined on the constants and predicates of the language such that: (1) for any constant a, $\mathbf{F}(a) \in \mathbf{D}$, and (2) for any predicate P^n, $\mathbf{F}(P^n) \subseteq \mathbf{D}^n$. In the usual way, we define an *assignment* to the variables (relative to model $\mathbf{M}$) to be any function $\mathbf{f}$ that maps each variable to an element of $\mathbf{D}$. And since our language contains complex terms, we define in the usual way by simultaneous recursion the *denotation* function for terms (relative to model $\mathbf{M}$ and assignment $\mathbf{f}$) and the *satisfaction* conditions for formulas (relative to model $\mathbf{M}$) (we shall suppress the subscripts that relativize these notions throughout). The denotation function shall be that function $\mathbf{d}(\tau)$ meeting the following conditions:[6]

1. Where a is any constant, $\mathbf{d}(a) = \mathbf{F}(a)$
2. Where x is any variable, $\mathbf{d}(x) = \mathbf{f}(x)$
3. Where $\imath x\varphi$ is any description,

$$\mathbf{d}(\imath x\varphi) = \begin{cases} \mathbf{o}\ (\in \mathbf{D}) \text{ iff } (\exists \mathbf{f}')(\mathbf{f}' \overset{x}{=} \mathbf{f}\ \&\ \mathbf{f}'(x) = \mathbf{o}\ \&\ \mathbf{f}' \text{ satisfies } \varphi \\ \quad \&\ (\forall \mathbf{f}'')(\mathbf{f}'' \overset{x}{=} \mathbf{f}'\ \&\ \mathbf{f}'' \text{ satisfies } \varphi \rightarrow \mathbf{f}'' = \mathbf{f}')) \\ \text{undefined, otherwise} \end{cases}$$

Notice that the clause governing descriptions packs, in a semantically precise way, Russell's analysis of definite descriptions into the conditions that must obtain if a description is to have a denotation. Notice also we do *not* assign some arbitrary object outside the domain of quantification to descriptions $\imath x\varphi$ for which φ fails to be uniquely satisfied; rather, we assign them nothing at all.

[6] In what follows, we use '$\mathbf{f}' \overset{x}{=} \mathbf{f}$' to mean that $\mathbf{f}'$ is an assignment function just like $\mathbf{f}$ except perhaps for what it assigns to x.

Now to complete the simultaneous recursion, we define $\mathbf{f}$ *satisfies* φ (i.e., φ is true (under $\mathbf{M}$) relative to assignment $\mathbf{f}$) as follows:

1. $\mathbf{f}$ satisfies $P^n\tau_1 \ldots \tau_n$ iff $\exists \mathbf{o}_1 \ldots \mathbf{o}_n \in \mathbf{D}((\mathbf{d}(\tau_1) = \mathbf{o}_1\ \&\ \ldots\ \&\ \mathbf{d}(\tau_n) = \mathbf{o}_n\ \&\ \langle \mathbf{o}_1, \ldots, \mathbf{o}_n \rangle \in \mathbf{F}(P^n))$
2. $\mathbf{f}$ satisfies $\tau = \tau'$ iff $\exists \mathbf{o}, \mathbf{o}' \in \mathbf{D}(\mathbf{d}(\tau) = \mathbf{o}\ \&\ \mathbf{d}(\tau') = \mathbf{o}'\ \&\ \mathbf{o} = \mathbf{o}')$
3. $\mathbf{f}$ satisfies $\neg\psi$ iff $\mathbf{f}$ fails to satisfy ψ
4. $\mathbf{f}$ satisfies $\psi \rightarrow \chi$ iff either $\mathbf{f}$ fails to satisfy ψ or $\mathbf{f}$ satisfies χ
5. $\mathbf{f}$ satisfies $\forall x\psi$ iff for every $\mathbf{f}'$, if $\mathbf{f}' \overset{x}{=} \mathbf{f}$, then $\mathbf{f}'$ satisfies ψ

Finally, in the usual way, we define: φ is true (under $\mathbf{M}$) iff every assignment $\mathbf{f}$ satisfies φ. φ is false (under $\mathbf{M}$) iff no $\mathbf{f}$ satisfies φ.

Notice that our definitions of satisfaction and truth are well-defined even for atomic and identity formulas that may contain non-denoting descriptions. Clauses 1 and 2 of the definition of satisfaction guarantee that an atomic or identity sentence will be true if and only if each term in the sentence has a denotation and the denotations stand in the right relationship.[7] Thus, atomic and identity sentences containing non-denoting descriptions are simply false. The truth conditions for the molecular and quantified sentences are the usual ones. Notice that from a semantic point of view, primitive descriptions occuring in molecular and quantified formulas are interpreted as if they have narrow scope. For example, the sentence $P\imath xQx \rightarrow S\imath xTx$ is true just in case: if there is a unique thing in $\mathbf{F}(Q)$ which is also in $\mathbf{F}(P)$, then there is a unique thing in $\mathbf{F}(T)$ which is also in $\mathbf{F}(S)$. It is a consequence that molecular sentences may contain non-denoting descriptions but nevertheless be true, since they may be true solely in virtue of their logical form. Thus, $P\imath xQx \rightarrow P\imath xQx$ is true regardless of whether the description $\imath xQx$ has a denotation.

We may associate with these semantic definitions a very simple logic for our language. All of the axioms and rules of classical propositional logic apply to the formulas, and all of the classical axioms and rules

[7] Note the trivial modification of the standard base clause in the definition of satisfaction which allows for the possibility that some terms may not have denotations. The standard base clause in the definition of satisfaction is: $\mathbf{f}$ satisfies $F^n\tau_1 \ldots \tau_n$ iff $\langle \mathbf{d}(\tau_1), \ldots, \mathbf{d}(\tau_n) \rangle \in \mathbf{F}(F^n)$. This clause would be undefined for descriptions that fail to denote. Our revised clause requires both that all terms have denotations and that the denotations stand in the right relationship for $\mathbf{f}$ to satisfy an atomic formula.

of predicate logic (with identity) apply to the constants and variables. However, the predicate logic of descriptions must be 'free', since these terms may fail to denote. This means that one cannot generalize upon a description $\imath x\varphi$ without first asserting $\exists y(y = \imath x\varphi)$. Specifically, one may not infer $\psi_z^{\imath x\varphi}$ from $\forall z\psi$ without first assuming $\exists y(y{=}\imath x\varphi)$, nor may one infer $\exists z\psi$ from $\psi_z^{\imath x\varphi}$ without this same assumption. Without such restrictions, one could infer $P\imath xQx$ directly from $\forall zPz$, for example. But such an inference would move one from truth to falsehood in models where everything is in the extension of the predicate 'P' and nothing (or more than one thing) is in the extension of the predicate 'Q.' Consequently, $\exists y(y{=}\imath xQx)$ is required before one may infer $P\imath xQx$ from $\forall zPz$.

It is important to recognize that *we could have avoided the use of free logic for descriptions*, by assigning some arbitrary denotation to descriptions $\imath x\varphi$ for which φ fails to be uniquely satisfied. It turns out that given the premises of the ontological argument described in the next section, we shall be able to prove that the description 'that than which none greater can be conceived' has a denotation.[8] Since this is the only description studied in this paper, and it provably has a denotation, we don't really have to make allowances for non-denoting descriptions as far as this paper is concerned.

So why, then, do we employ free logic for descriptions? Mainly for psychological reasons. By using free logic, we remove any suspicion that it is the models of our language, or the semantic definition of denotation, which force descriptions to have a denotation. We want our readers to be assured that it is not a formal device embodied by the very logical set-up that guarantees that the description 'that than which none greater can be conceived' has a denotation. Any demonstration that this description does have a denotation will not be a matter of logic alone, but must depend on additional non-logical premises.

Moreover, we would like to stress the fact that our use of free logic does not commit us to any arguments used to justify this logic. Indeed, we explicitly reject a certain argument that free logicians have used to conclude that the logic of constants must be free. They argue, from the fact that in standard quantification theory it is a theorem that $\exists y(y{=}a)$, for any constant a, that the *existence* of the things denoted by the constants becomes a matter of logic. But, of course, in a logic that separates being (or quantification) from existence, the only thing that the theorem in question shows is that the things denoted by the constants have being, not that they exist. This simply means that we can talk about and quantify over the thing denoted by 'a' without presupposing that this thing exists.

[8] This is not to be confused with the proof that the object denoted by the description *exists*, which we take to be the main point of the Ontological Argument—recall we are distinguishing $\exists y(y{=}\imath x\varphi)$ from $E!\imath x\varphi$.

Finally, note that a single logical axiom governs the behavior of definite descriptions. Where ψ is an atomic formula or identity formula in which z occurs free, the axiom governing descriptions may be formulated as follows:

$$\textit{Description Axiom: } \psi_z^{\imath x\varphi} \leftrightarrow \exists y(\varphi_x^y \;\&\; \forall u(\varphi_x^u \rightarrow u{=}y) \;\&\; \psi_z^y)$$

To see what this says, consider the following instance, in which ψ is 'Rz' and $\imath x\varphi$ is '$\imath xQx$':

$$R\imath xQx \leftrightarrow \exists y(Qy \;\&\; \forall u(Qu \rightarrow u{=}y) \;\&\; Ry)$$

This simply says: the thing which is Q is R iff there is something y such that (a) y is Q, (b) everything that is Q is identical to y, and (c) y is R. This axiom expresses in our language Russell's analysis of the definite description without eliminating descriptions from the language.[9]

[9] The restriction on the *Description Axiom* merely reflects the fact that the correspondence theory of truth applies only to logically simple formulas! The correspondence theory of truth, which presumes truth to be a kind of correspondence between language and the world, places the following constraint on the truth of atomic and identity formulas φ: φ is true if and only if every term in φ has a denotation and the denotations stand in the correct relationships. Otherwise, how could an atomic or identity formula having a non-denoting term be *true* in virtue of some feature of the world? Note that this constraint doesn't apply to logically complex formulas. Consider a molecular formula φ, say $\psi \rightarrow \chi$, where ψ is an atomic formula having a non-denoting term. φ is true, because the antecedent is false (given the constraints of the correspondence theory and two-valued logic). Complex formulas, even those containing non-denoting terms, may be true in virtue of their logical form. So the constraint doesn't apply to logically complex formulas. The restriction on the *Description Axiom* simply acknowledges this fact.

An example we've already considered makes this vivid. Note that the formula '$Pz \rightarrow Pz$' is a logical truth, and remains a logical truth even when 'z' is replaced uniformly by a non-denoting description, say '$\imath xQx$.' Without the restriction on the *Description Axiom*, the following biconditional would be an instance:

$$(P\imath xQx \rightarrow P\imath xQx) \leftrightarrow \exists y[Qy \;\&\; \forall u(Qu \rightarrow u{=}y) \;\&\; (Py \rightarrow Py)]$$

However, for models in which '$\imath xQx$' doesn't denote, this biconditional is *false*, since the left side would be true (in virtue of its logical form) while the right side would be false.

To state our first important theorem governing descriptions, let '$\exists! y\varphi$' be an abbreviation for '$\exists y \forall u(\varphi^u_y \leftrightarrow u = y)$.' Thus, we may read '$\exists! y\varphi$' as 'there is a unique y such that φ.' Note that the following is a simple consequence of the *Description Axiom*:

Description Theorem 1: $\exists! x\varphi \rightarrow \exists y(y = \imath x\varphi)$[10]

Semantically, *Description Theorem 1* tells us that if condition φ is uniquely satisfied, then the description "the x such that φ" is guaranteed to have a denotation. So if one knows that a condition φ is uniquely satisfied, one may *introduce* a description to denote the thing that uniquely satisfies it. It will soon become clear that this is something that Anselm does implicitly in his argument. Moreover, this theorem tells us that such descriptions may be generalized—they can be instantiated into universal claims or replaced by existentially quantified variables.

The following lemma is also a consequence of the *Description Axiom*:

Lemma 1: $\tau = \imath x\varphi \rightarrow \varphi^\tau_x$, for any term τ.[11]

Given *Lemma 1*, it should be easy to see that the following is a theorem:

Description Theorem 2: $\exists y(y = \imath x\varphi) \rightarrow \varphi^{\imath x\varphi}_x$

Proof: Assume the antecedent, namely, $\exists y(y = \imath x\varphi)$. So there must be some object, say c, such that $c = \imath x\varphi$. But by Lemma 1, it follows that φ^c_x. So $\varphi^{\imath x\varphi}_x$. ⋈

Here is a simple instance of this schema: $\exists y(y = \imath xQx) \rightarrow Q\imath xQx$. In other words, if there is something that is *the* Q-thing, then it (i.e., the Q-thing) must have the property Q. We believe that this simple theorem plays a central role in the Ontological Argument.

So without the restriction, the *Description Axiom* would constrain the descriptions in true complex formulas to have denotations. That is, without the restriction, the *Description Axiom* extends the constraints of the correspondence theory to logically complex formulas. This, as we saw in the previous paragraph, can not be done.

[10] *Proof:* Assume the antecedent, that is: $\exists x \forall u(\varphi^u_x \leftrightarrow u = x)$. So there must be some object, say a, such that: $\forall u(\varphi^u_x \leftrightarrow u = a)$. And since $a = a$, it follows that φ^a_x. But we now know: φ^a_x & $\forall u(\varphi^u_x \rightarrow u = a)$ & $a = a$. So by Existential Generalization, this yields: $\exists y(\varphi^y_x$ & $\forall u(\varphi^u_x \rightarrow u = y)$ & $a = y)$. And by the *Description Axiom*, it follows that $a = \imath x\varphi$. ⋈

[11] *Proof:* Assume the antecedent. Now if we let ψ be '$\tau = z$', our antecedent has the form $\psi^{\imath x\varphi}_z$. So by the Description Axiom, it follows that: $\exists y(\varphi^y_x$ & $\forall u(\varphi^u_y \rightarrow u = y)$ & $\tau = y)$. So, there must be some object, say b, such that: φ^b_x & $\forall u(\varphi^u_y \rightarrow u = b)$ & $\tau = b$. If so, then, it follows that: φ^τ_x. ⋈

Non-Logical Predicates and Meaning Postulates

In order to represent the premises of Anselm's argument, we must add to our formal language some non-logical predicates and meaning postulates. Since our view is that the argument turns on the distinction between being and existence, we begin by discussing how this distinction is to be represented. Our plan is simply to add the special predicate '$E!$' to denote the property of *existence*. Note that there is a difference between formulas of the form '$\exists x\varphi$' and formulas of the form '$\exists x(E!x$ & $\varphi)$'. We read the former as "there is an x such that φ," or "some x is such that φ." We read the latter as "there is an x having the property of *existence* which is such that φ," or "there *exists* an x such that φ." In other words, we are not reading the quantifier '$\exists$' as existentially loaded. Unfortunately, there is a tradition of calling '$\exists$' the 'existential' quantifier, and in what follows, we conform to that tradition. But we want to make it clear that we do not use the existential quantifier to assert existence.[12]

We therefore absolutely reject the definition of '$E!x$' as '$\exists y(y = x)$'. This definition would collapse the very distinction that proves crucial to the argument. Indeed, the rejection of this definition has led to some rather interesting and exciting new developments in metaphysics. Recently, Terence Parsons ([1974], [1979], and [1980]) has developed a precise and fruitful new theory of nonexistent objects. Parsons' theory asserts that there are objects that don't exist, and this assertion is captured formally as: $\exists x(\neg E!x)$. Now if '$E!x$' were defined as '$\exists y(y = x)$', Parsons' theory would assert a manifest logical falsehood, namely: $\exists x \neg \exists y(y = x)$.[13] So, in his metaphysical framework, the distinction between quantifying over x and predicating existence of x, reflecting the difference between being and existence, is crucial. Parsons' theory offers clearcut responses to the standard objections to theories of nonexistent objects and has interesting applications, which include the problem of negative singular existential statements and the problem of analyzing statements in and about fiction.

[12] We should also remind the reader to distinguish the symbols '$\exists!$' from '$E!$'. The former is used as a quantifier that asserts *uniqueness*, whereas the latter is just the existence predicate.

[13] The language and semantics we have developed is therefore similar to Parsons' in the following respects: '$\exists x(\neg E!x)$' may be consisently asserted (in formal terms, this formula is satisfiable), and '$\exists x \neg \exists y(y = x)$' is false in every model (and thus a logical falsehood).

In addition to Parsons' work, one of the present authors has developed a metaphysical theory of abstract objects that also utilizes the distinction between the quantifier and the existence predicate. In Zalta [1983] and [1988], one finds '$E!$' used as a predicate denoting the property of existence. An object x is defined to be *abstract* ($A!x$) iff x couldn't possibly exemplify the property of existence ($\neg\Diamond E!x$). Zalta's metaphysical theory entails that there are such abstract objects ($\exists x A!x$), which in turn entails that there are objects that don't exist ($\exists x \neg E!x$). Zalta's abstract objects are then used to model such things as monads, possible worlds, fictional characters, Fregean senses, and mathematical objects, as part of the applications of the theory.

There are two points to be made by referring to the work of these authors. The first is that their work demonstrates that distinguishing being from existence is not only coherent but also useful. If Parsons and Zalta are right, then that difference underlies and explains many of our commonsense beliefs. The second is that, whether or not one accepts their theories of objects, the metaphysical framework they presuppose, that of a realm of objects about which one can talk and over which one can quantify *whether or not they exist*, is very similar to the framework Anselm presupposes. Here is why.

Inspection of Anselm's language in *Proslogium* II reveals that he explicitly contrasts objects that have being in the understanding (*esse in intellectu*) with those that have being in reality (*esse in re*) as well. Anselm sometimes refers to these two kinds of being as 'existence in the understanding' and 'existence in reality'. For example, line 12 of *Proslogium* II begins with '*Existit*', as opposed to '*Esse*', and concludes that God exists both in the understanding and in reality.[14] Let us, just for the moment, continue to speak with Anselm, and use the phrases 'being in the understanding' and 'exists in the understanding' interchangeably (and similarly for 'being in reality' and 'exists in reality'). Note also that Anselm speaks as if one and the same thing can have being (exist) either solely in the understanding, or can simultaneously have being (exist) both in the understanding and in reality. This is presupposed by his view that it is greater to have being (exist) both in the understanding and in reality than to have being (exist) in the understanding alone.

In summary, Anselm believes that there is a difference between the two

[14]The reader may wish to examine the Appendix, where the original Latin and a line by line translation (by Barnes [1972]) may be found.

metaphysical states of being in the understanding and being in reality. His use of the verbs 'to be' and 'to exist' is unregimented. He relies, rather, on the qualifying phrases 'in the intellect' and 'in reality' to reflect that difference. Parsons and Zalta rely on a regimented use of 'there is' and 'there exists' to reflect a similar difference.

Now if we were to represent Anselm's manner of speaking in a strict way, we would need three distinct notions. A quantifier '$\exists$' would be needed to represent Anselm's notion 'there is' (*esse*) or 'there exists' (*existit*); a predicate 'U' would be needed to represent the property of being in the understanding; a predicate '$E!$' would be needed to represent the property of being in reality. We would then be able to represent Anselm's quantification over two kinds of objects by the distinction between '$\exists x(Ux \ \& \ \ldots)$' and '$\exists x(E!x \ \& \ \ldots)$'. The former reads 'there is (exists) an x in the understanding such that ...', while the latter reads 'there is (exists) an x in reality such that ...'.

However, for reasons that will become plain, we shall simplify the formula '$\exists x(Ux \ \& \ \varphi)$' to '$\exists x\varphi$', and read the latter as 'there is an x such that φ', where this is to be distinguished from 'there exists an x such that φ' ('$\exists x(E!x \ \& \ \varphi)$'). We thus assimilate Anselm's language to that used by Parsons and Zalta, for Anselm's notion of 'being (existence) in the understanding' corresponds to Parsons' and Zalta's notion of being or quantification, and Anselm's notion of 'being (existence) in reality' corresponds to Parsons' and Zalta's notion of existence. These correspondences establish an analogy between the metaphysical picture presupposed by Anselm, on the one hand, and that described by Parsons and Zalta, on the other. Both pictures include two realms of objects, one having a greater degree of reality than the other. In both pictures, an object can "inhabit" both realms simultaneously: Anselm concludes on line 12 of *Proslogium* II that God exists both in the intellect and in reality; for Parsons and Zalta, the existence of an object x ($E!x$) entails the being of x ($\exists y \ y = x$). Moreover, in both frameworks, one can talk about an object x, predicate things of x, and quantify over x, regardless of whether x inhabits one or both of the ontological realms.

So by adopting Parsons' and Zalta's regimented use of 'there is' and 'there exists' and letting '$\exists x\varphi$' go proxy for '$\exists x(Ux \ \& \ \varphi)$' we shall be able to simplify the formulas that will be used in the argument. As the reader will soon be able to verify, this simplification has no untoward consequences. For our reconstruction of Anselm's argument, the notion

of 'being' works just as well in opposition to the notion of 'existence (in reality)' as does the notion of 'being in the understanding'. However, if perfect faithfulness to Anselm is desired, then it is a routine exercise to add the conjunct 'Ux' in the appropriate places of the argument that we present in the next section.

We turn, then, to an analysis of the premises of Anselm's ontological argument. These premises require the following special predicates and meaning postulate. First, 'C' is to be a one-place predicate and 'Cx' is to be read as: x can be conceived. Second, 'G' is to be a two-place predicate and 'Gxy' is to be read as: x is greater than y. For reasons that will become apparent, the relation denoted by 'G' must be *connected*. In other words, G obeys the following meaning postulate (which constitutes a non-logical axiom): $\forall x \forall y(Gxy \lor Gyx \lor x = y)$. We shall discuss this requirement in greater detail in the next section.

The Premises of the Argument

We believe that the first premise of Anselm's main argument in the *Proslogium* II is: there is (in the understanding) something than which nothing greater can be conceived. This premise doesn't occur until line 8. In lines 1 through 7, Anselm works his way through a subargument designed to marshal agreement on this claim. He concludes: "Therefore even the fool is bound to agree that there is at least in the understanding something than which nothing greater can be imagined"[15] We believe that the conclusion of the subargument in lines 1 through 7, when stripped of the operator "even the fool is bound to agree that," serves as the first premise of the Ontological Argument.

From our discussion in the previous section, it should be clear that a strict representation of Anselm's words would be: $\exists x(Ux \,\&\, \neg\exists y(Gyx \,\&\, Cy))$. Although this is a perfectly good representation of the first premise, we have found that it is more elegant to formulate the first premise as follows:

Premise 1: $\exists x(Cx \,\&\, \neg\exists y(Gyx \,\&\, Cy))$

Premise 1 simply asserts that there is a conceivable thing which is such that nothing greater can be conceived. There is one basic difference between this formulation and the former one: the clause 'Cx' replaces 'Ux'

[15] See the Appendix.

as the first conjunct of the quantified claim. Given our discussion in the previous section, it should be clear why we have dropped the conjunct 'Ux' from our representation of this premise. And little justification is needed for adding the clause 'Cx'. It makes explicit what is implicit in the clause 'nothing greater can be conceived,' namely, that any such object is itself conceivable.

Further evidence that our formulation of *Premise 1* with the formula '$Cx \,\&\, \neg\exists y(Gyx \,\&\, Cy)$' is a good one comes from the fact that it now follows, by the non-logical axiom for *greater than* alone, that if something satisfies this formula, then something *uniquely* satisfies this formula. In other words, if there is some conceivable thing such that nothing greater can be conceived, there is a *unique* conceivable thing such that nothing greater can be conceived. To see this, let us use 'φ_1' to abbreviate the open formula '$Cx \,\&\, \neg\exists y(Gyx \,\&\, Cy)$.' Now consider the following lemma:[16]

Lemma 2: $\exists x\varphi_1 \rightarrow \exists! x\varphi_1$

Proof: If we assume the antecedent, all we have to prove is that at most one thing satisfies φ_1. So assume: $\exists x(Cx \,\&\, \neg\exists y(Gyx \,\&\, Cy))$. So there must be some object, say a, such that: $Ca \,\&\, \neg\exists y(Gya \,\&\, Cy)$. Recall that all we have to show is that no other conceivable thing distinct from a is such that nothing greater can be conceived. Suppose, for *reductio*, that: $\exists z(z \neq a \,\&\, Cz \,\&\, \neg\exists y(Gyz \,\&\, Cy))$. Let us call such an object 'b.' We derive a contradiction from this as follows. We know, by the non-logical axiom governing G that $Gab \lor Gba \lor a = b$. Since $b \neq a$, either $Gab \lor Gba$. But both disjuncts lead to contradiction. Suppose Gab. Then, since Ca, it follows that: $Gab \,\&\, Ca$. So $\exists y(Gyb \,\&\, Cy)$, contrary to the *reductio* hypothesis. But then suppose Gba. Again, since by hypothesis Cb, it follows that: $Gba \,\&\, Cb$. Again, it follows that $\exists y(Gya \,\&\, Cy)$, contrary to our initial assumption. By *reductio*, then, it follows that $\neg\exists z(z \neq a \,\&\, Cz \,\&\, \neg\exists y(Gyz \,\&\, Cy))$. ⋈

It would serve well to explain semantically just why it is that *Lemma 2* is true. To do this, let us consider an arbitrary model that makes the antecedent of *Lemma 2* true and see why it is that the consequent of *Lemma 2* must also be true. In any model of the antecedent of *Lemma 2*,

[16] If you prefer to represent *Premise 1* as '$\exists x(Ux \,\&\, \varphi_1)$,' then let φ_2 be '$Ux \,\&\, \varphi_1$', and replace φ_1 by φ_2 everywhere in what follows.

there must be a set of objects called the *conceivable* objects amongst the members of which the *greater than* relation sometimes holds. To picture these models, suppose that an arrow points *away* from conceivable object a to conceivable object b whenever a is greater than b. Now, in any model in which '$\exists x\varphi_1$' is true, i.e., in which there is a conceivable object such that no conceivable object is greater, there has to be at least one object *having no arrows pointing towards it*! Such an object is called a 'maximal element,' and of course, there may be several such maximal elements. Let us suppose that there are indeed several. Now the model also has to make the non-logical axiom expressing the connectedness of the *greater than* relation true as well. So anytime you find a pair of maximal elements a and b, where $a \neq b$, *connectedness* requires that an arrow go from a to b or b to a or both. Note that there can't be *both* an arrow going from a to b and from b to a, for otherwise, a and b would both cease to be maximal (the antecedent of *Lemma 2* requires that there be at least one maximal element). Consequently, either an arrow goes from a to b or from b to a (but not both), and so at most one of these two elements will be a maximal element (having no arrows pointing towards it). Since this reasoning applies to any pair of candidate maximal elements, it follows that there is a unique element having no arrows pointing towards it. Thus, the consequent of *Lemma 2* is true, that is, there is a unique conceivable object such that no conceivable object is greater.[17]

Another interesting fact about *Lemma 2* is that its truth doesn't require that *greater than* order the conceivable objects. Recall that a relation R *partially orders* a set S just in case R is anti-symmetric and transitive on the members of S.[18] R *totally orders* S whenever R partially orders S and R is connected. But a connected relation R doesn't have to be either transitive or anti-symmetric. To see this, consider the relation $R = \{\langle a, b\rangle, \langle b, a\rangle\}$. R is connected, but to make it transitive, you need to add the pair $\langle a, a\rangle$; to make it anti-symmetric, you have to delete either the pair $\langle a, b\rangle$ or the pair $\langle b, a\rangle$.

Now we believe that only one other premise is used in the Ontological Argument. It occurs in the following line of *Proslogium* II: "For if it is at

[17] In this semantic proof, we began with a model in which the antecedent of *Lemma 2* is true, and looked at what happened when the connectedness of *greater than* is added. Alternatively, we might have started with a model in which *greater than* is a connected relation, and then looked at what happens when you add the antecedent of *Lemma 2*. This is an exercise left for the reader.

[18] R is *anti-symmetric* $=_{df}$ $\forall x \forall y(x \neq y \rightarrow (Rxy \rightarrow \neg Ryx))$

least in the understanding alone, it can be imagined to be in reality too, which is greater." One way of expressing Anselm's point here as follows: if that than which none greater can be conceived doesn't exist (in reality), then something greater than it can be conceived. In order to represent this premise formally, note that the description '$\imath x\varphi_1$' is the proper translation of "that (conceivable thing) than which none greater can be conceived." We now take the following to be *Premise 2* of the argument:

Premise 2: $\neg E!\imath x\varphi_1 \rightarrow \exists y(Gy\imath x\varphi_1 \;\&\; Cy)$

In our interpretation of this premise, we don't explicitly identify the object that satisfies the matrix quantified in the consequent. Anselm, however, identifies that object as one just like the nonexistent object of the antecedent but existing. Our slightly weaker interpretation suffices for the argument.

It is worthwhile to note here how *Lemma 2* and *Description Theorem 1* connect *Premises 1* and *2*. From *Premise 1* and *Lemma 2*, it follows that $\exists! x\varphi_1$. From this and *Description Theorem 1*, it follows that $\exists y(y = \imath x\varphi_1)$. This chain of reasoning shows that there is such a thing as *the* thing than which none greater can be conceived. Such a proof that '$\imath x\varphi_1$' has a denotation justifies the introduction and use of this description in any argument based on *Premise 1*. In particular, it justifies the introduction and use of the description in *Premise 2*.

Finally, let us define:

D_1 God ('g') $=_{df}$ $\imath x\varphi_1$

So, with Anselm, we have defined 'God' to be "the conceivable thing than which no greater can be conceived." One might think that definition D_1 introduces the constant 'g' as a mere abbreviation of the definite description. However, if the constant 'g' is utilized in an argument *after* it is shown that the description '$\imath x\varphi_1$' has a denotation (for example, as shown in the previous paragraph), then D_1 doesn't simply provide a way to abbreviate the description, but rather introduces the logical fact that 'g' is a genuine constant that has the same denotation as the description.

The Ontological Argument

From *Premise 1*, it follows that $\exists! x\varphi_1$, by *Lemma 2*. From this, it follows that $\exists y(y = \imath x\varphi_1)$, by *Description Theorem 1*. From this, it follows that: $C\imath x\varphi_1 \;\&\; \neg\exists y(Gy\imath x\varphi_1 \;\&\; Cy)$, by *Description Theorem 2*.

Now, for *reductio*, assume: $\neg E!\imath x\varphi_1$. Then, by *Premise 2*, it follows that $\exists y(Gy\imath x\varphi_1 \ \& \ Cy)$, which is a contradiction. So $\neg\neg E!\imath x\varphi_1$, i.e., $E!\imath x\varphi_1$. And by D_1, it follows that $E!g$, i.e., God exists. $\bowtie$

Features of Our Interpretation

This is a valid argument. It is simple, elegant, and faithful to the text. Our reading has certain virtues not found in other readings in the literature, and in particular, the readings offered by Plantinga, Lewis, Adams, and Barnes. Though Plantinga, Lewis, and Adams do distinguish between quantification and existence, modal inferences play a central role in their readings, and none of these authors takes definite descriptions seriously. Of the four, Barnes is the only one to have eliminated modality from the argument, though we think that his treatment of the definite description is not as satisfying as ours.[19] Here, then, is a list of some new and interesting features of our interpretation of the argument.

First, the validity of our interpretation doesn't depend on the validity of modal inferences. It depends solely on the distinction between being (quantification) and existence, the logic of descriptions, and two premises having at least some *prima facie* plausibility. We represent the phrase 'that than which none greater can be conceived' as a definite description, which is what Anselm took the phrase to be.

Second, our interpretation reveals that *Description Theorem 2* is crucial to the argument. If the description '$\imath x\varphi$' denotes, then one may legitimately substitute it for each free x in φ, no matter how complicated φ is. This principle of 'turning the description upon itself' makes the *reductio* argument possible.

Third, our interpretation *justifies* Anselm's introduction of a definite description into the argument. That is, his first premise is simply that there is *something* such that nothing greater can be conceived. But then he begins talking in terms of the description "that than which none greater can be conceived." *Lemma 2* and *Description Theorem 1* together justify this move.

Fourth, our interpretation reveals that the connectedness of the *greater than* relation suffices for the purposes of the argument. The appeal to the connectedness of *greater than* is made in *Lemma 2*. So *greater than* doesn't have to order the conceivable objects! This is quite an unexpected result. Besides being connected, the only constraint on *greater than* is that it be a relation that makes *Premises 1* and *2* true.

Fifth, our interpretation seems to make better sense of the commentary on the argument by St. Thomas Aquinas. Aquinas took Anselm to be arguing that the existence of God is obvious.[20] Our reading shows how very simple Anselm's argument is, yet if it had relied on a lot of modal technicalities, then Aquinas would hardly have thought that Anselm takes the existence of God to be self-evident.

Sixth, unlike modal interpretations, our reading makes sense of Kant's criticisms of the argument. In the first *Critique*, Kant argues that 'existence' is not an (analytical) predicate of a thing.[21] It doesn't really matter, for the present purposes, whether one believes that Kant was right or wrong about this. What matters is that his criticism presupposes that the argument is formulated with 'existence' as a predicate. On our conception, the argument is so formulated.

Last, and most interesting of all, on our interpretation, the argument may be transformed into an argument schema for conclusions of the form *God exemplifies* F, for any perfection F. To see how, note that one can substitute for the existence predicate in *Premise 2* any predicate that denotes a perfection and the premise remains true. For example, if you let F be *omnipotence*, then *Premise 2* asserts: If $\imath x\varphi_1$ is not omnipotent, then something greater than $\imath x\varphi_1$ can be conceived. Now the argument, and in particular, the *reductio*, will proceed in the same way. The conclusion will be, God exemplifies F, no matter which perfection F is used in the argument. We take this to be an insight into the structure of the ontological argument.[22]

This last feature of our reading establishes a link between Anselm's argument and Descartes' argument in *Meditation V*. Recall that one of Descartes' arguments in *Meditation V* is a deduction of God's existence from the claim that God is a being with every perfection and the claim that existence is a perfection. But, to get this result, it looks as though Descartes has to *define* God to be that being having every perfection,

[19] Compare his proof (Barnes, [1972], p. 88–9) with ours, and especially his Step 26 with our use of *Description Theorem 2*. Note also that *five* assumptions are required in his reading of the argument—though in all fairness it must be said that he assimilates both the subargument for Premise 1 and the main argument into a single argument.

[20] See Aquinas' *Summa Theologica*, Book 1, Question 2, Article 1, Second Objection.

[21] See Kant's *Critique of Pure Reason*, translated by F. Max Müller, London: Macmillan, 1896, Chapter III, §4 of the Transcendental Dialectic.

[22] The authors are indebted to Harry Deutsch for noticing that our argument is generalizable.

whereas Anselm's argument appears to be general enough for us to *derive*, for every perfection F, that God has F. Thus, defining 'God' the way Anselm does allows us to derive facts about God which for Descartes have to be assumed in the definition. This puts Anselm's and Descartes' arguments in better perspective. Philosophers have often wondered whether Descartes' definition of 'God' as 'that Being having all perfections' is equivalent to Anselm's definition as 'that than which none greater can be conceived.' Our answer is 'No!,' for the above considerations suggest that Descartes' definition is derivable from Anselm's, but not vice versa. It appears that the only way to derive Anselm's definition from Descartes' is to add additional hypotheses.

The above considerations suggest that our interpretation has much to offer philosophers interested in Anselm, in the history of philosophy, and in the nature of the ontological argument in general. Before we conclude, however, it is instructive to enquire why this conception of the argument has just now surfaced. The reason is that it has only been in the past ten years that logical systems have been constructed in which the logic of descriptions and the distinction between being and existence have been combined. After [1905], when Russell introduced his method of eliminating descriptions, few logicians took descriptions seriously as genuine terms. In [1956], H. Leonard began the investigation of free logic, and descriptions began to be taken seriously as genuine terms. They were analyzed as terms "free of existence assumptions." But the free logicians followed Russell's [1908] assumption that the formula '$E!\imath x\varphi$' is just equivalent to the formula '$\exists x\forall y(\varphi^y_x \leftrightarrow y = x)$.'[23] For example, this equivalence appears as a valid law in H. Leonard's [1956].[24] Consequently, these logicians just identified 'being' and 'existence,' and read the formula '$\exists x\varphi$' as "there exists an x such that φ." But in [1980], T. Parsons developed a coherent system of *nonexistent* objects, in which it makes sense to say "there *are* things that don't *exist*", or '$\exists x\neg E!x$.' Moreover, in Parsons' system, descriptions are taken seriously as genuine terms, so that they may denote objects even though the objects denoted do not exist; '$\exists y(y = \imath x\varphi)$' and '$E!\imath x\varphi$' express different things. We believe that this kind of formal system holds the key to the logic of Anselm's ontological argument. It is, therefore, a recent innovation in intensional logic that

[23] See Russell [1908], p. 93.

[24] See Leonard [1956], p. 60, Law L4.

has made our reading of the argument possible.[25]

In the *Proslogium*, St. Anselm meditates on how we can be sure by natural reason alone that God exists. His focus is on the eminence of God. Everyone has the idea of that than which no greater can be conceived. In

[25] Actually, Parsons has a section on the ontological argument in [1980] (pp. 212–217), but nothing like our interpretation of the argument is to be found there. Instead, in footnote 1 (p. 214), he discusses the reading of the argument presented in Barnes [1972]. Parsons then goes on to say that a *de re*/*de dicto* ambiguity undermines Anselm's argument (p. 215). He says:

> Anselm's argument begins by establishing that the fool 'imagines that than which nothing greater can be imagined' in its *de dicto* sense (for the justification is merely that the fool understands the words). But then he begins referring back to the alleged referent of the denoting phrase by means of singular pronouns, as if it had been established that there is such a object imagined by the fool (*de re*), a natural and reasonably subtle transition—but a question-begging one.

We don't believe that Anselm does commit the fallacy Parsons attributes to him in this passage. We have shown that Anselm is *justified* in using the anaphoric singular pronouns once he asserts *Premise 1*, for by *Lemma 2*, it follows that there is a unique object such that nothing greater can be conceived. So given that Anselm believes *Premise 1* to be true, his use of anaphoric pronouns is legitimate.

Parsons' objection is best taken not as directed at *Premise 1* but rather at Anselm's support for *Premise 1*. Here is what appears to be Anselm's subargument for *Premise 1*, where ψ stands for the phrase "none greater can be conceived":

> *Premise*: Anyone (even a fool) can understand the phrase "that than which ψ."
>
> *Premise*: If anyone (even a fool) can understand the phrase "that than which ψ," there is something in the understanding such that ψ.

These two premises entail *Premise 1*. Parsons' complaint concerns the *second* premise of this subargument, namely, that its consequent makes a *de re* claim although its antecedent is grounded on *de dicto* considerations. The antecedent asserts that anyone can understand a certain denoting phrase. The consequent, however, is a quantified *de re* claim the truth of which requires that there be a certain object (*res*) in the understanding. Parsons might argue that understanding a denoting phrase doesn't entail any *de re* claims. However, it seems certain that Anselm felt justified in thinking that to understand a phrase (even *de dicto*), there must be something in the understanding, such as an idea, that is grasped. Nevertheless, Parsons may be raising a legitimate question about whether there is a thing in the understanding which is such that ψ and which is grasped when the phrase "that than which ψ" is understood. The answer to this question depends on one's analysis of the intentionality of directed mental states and the intensionality of denoting phrases. Anselm has a philosophy of language, knowledge, and mind that provides such analyses in general, and in particular, how God's being in the fool's understanding follows from the fool's saying "there is no God." A discussion of these issues would take us beyond the scope of this paper. Consequently, any further comment on Anselm's support for *Premise 1* must be postponed.

order not to prejudice the issue, he doesn't assume that everything that is exists. Some things can be merely in the understanding, without existing in reality as well. This is just to avoid assuming the desired conclusion. One of the things we show is that a very simple point about the logic of descriptions, namely *Description Theorem 2*, and a simple point about *greater than*, namely, that it is connected, is all the technical apparatus Anselm really needs. Anselm's argument for the existence of God doesn't depend on a sophisticated theory about multiple possible worlds. The logical mechanisms and metaphysical assumptions of Anselm's *Proslogium* II argument are paradigms of simplicity.

Appendix: Anselm's *Proslogium* II

1. Therefore, Lord, who grant understanding to faith, grant me that, in so far as you know it beneficial, I understand that you are as we believe and you are that which we believe. (*Ergo, Domine, qui das fidei intellectum, da mihi, ut, quantum scis expedire, intelligam quia es, sicut credimus; et hoc es, quod credimus.*)

2. Now we believe that you are something than which nothing greater can be imagined. (*Et quidem credimus te esse aliquid, quo nihil majus cogitari possit.*)

3. Then is there no such nature, since the fool has said in his heart: God is not? (*An ergo non est aliqua talis natura, quia* dixit insipiens in corde suo: Non est Deus?)

4. But certainly this same fool, when he hears this very thing that I am saying — something than which none greater can be imagined — understands what he hears; and what he understands is in his understanding, even if he does not understand that it is. (*Sed certe idem ipse insipiens, cum audit hoc ipsum quod dico, aliquid quo majus nihil cogitari potest; intelligit quod audit, et quod intelligit in intellectu ejus est; etiamsi non intelligat illud esse.*)

5. For it is one thing for a thing to be in the understanding and another to understand that a thing is. (*Aliud est enim rem esse in intellectu; aliud intelligere rem esse.*)

6. For when a painter imagines beforehand what he is going to make, he has in his understanding what he has not yet made but he does not yet understand that it is. (*Nam cum pictor præcogitat quæ facturus est, habet quidem in intellectu; sed nondum esse intelligit quod nondum fecit.*)

7. But when he has already painted it, he both has in his understanding what he has already painted and understands that it is. (*Cum vero jam pinxit, et habet in intellectu, et intelligit esse quod jam fecit.*)

8. Therefore even the fool is bound to agree that there is at least in the understanding something than which nothing greater can be imagined, because when he hears this he understands it, and whatever is understood is in the understanding. (*Convincitur ergo etiam insipiens esse vel in intellectu aliquid, quo nihil majus cogitari potest; quia hoc cum audit, intelligit; et quidquid intelligitur, in intellectu est.*)

9. And certainly that than which a greater cannot be imagined cannot be in the understanding alone. (*Et certe id, quo majus cogitari nequit, non potest esse in intellectu solo.*)

10. For if it is at least in the understanding alone, it can be imagined to be in reality too, which is greater. (*Si enim vel in solo intellectu est, potest cogitari esse et in re; quod majus est.*)

11. Therefore, if that than which a greater cannot be imagined is in the understanding alone, that very thing than which a greater cannot be imagined is something than which a greater can be imagined. But certainly, this cannot be. (*Si ergo id, quo majus cogitari non potest, est in solo intellectu, id ipsum, quo majus cogitari non potest, est quo majus cogitari potest: Sed certe hoc esse non potest.*)

12. There exists, therefore, beyond doubt something than which a greater cannot be imagined, both in the understanding and in reality. (*Existit ergo procul dubio aliquid quo majus cogitari non valet, et intellectu, et in re.*)

translated by Jonathan Barnes

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